Working Paper

Alternative Real Assets in a Portfolio Context

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In this article we are introducing the asset class "Alternative Real Assets". This asset class provides access to "real" investment opportunities, real meaning both partially inflation protected and solid, via limited partnership fund structures. This (sub-)asset class comprises fund vehicles that invest, for example, in infrastructure, shipping, or renewable energy projects. For some of these alternative real assets, financial incentives are provided, such as preferential feed-in tariffs for solar-generated electricity.

We discuss alternative real assets, including a comparison to other asset classes and a description of distinctive features, with a focus on photovoltaic investments. The bottom-up modeling approach of individual photovoltaic projects, and the approach to incorporate these investments in portfolio analytics, are presented. Based on a comprehensive framework, we are finally able to evaluate the benefits of photovoltaic investments in a multi-asset portfolio.

Investing in photovoltaic facilities is environmentally supportive. We are able, as well, to show that under reasonable assumptions, financing solar plants is an attractive asset class, both stand-alone and in a portfolio context.

1.1. Introduction

This article introduces the asset class "Alternative Real Assets". Alternative real assets provide access to investment opportunities in special forms of project finance, usually via limited partnership structures. This asset class comprises fund vehicles that invest, for example, in infrastructure, shipping, or renewable energy projects. For some of these alternative real assets, financial incentives are in place, such as preferential feed-in tariffs for solar-generated electricity.

The analysis of photovoltaic investments is the focus of this article. A bottom-up modeling of investments in photovoltaic facilities, as well as the approach to incorporate these investments in portfolio analytics, is presented. Based on a comprehensive framework, we are able to evaluate the benefits of photovoltaic investments in a multi-asset portfolio. Empirical data and realistic modeling assumptions are the result of a joint research project with a leading provider of closed-end funds in the area of photovoltaic investments.

This article is organized as follows. Section 2 provides an introduction to alternative real assets. In Section 3 we present our bottom-up approach to model alternative real asset investments. Section 4 describes how to incorporate these investments to a portfolio. It shows the results of integrating photovoltaic investments into a traditional portfolio, including changes in portfolio statistics and the corresponding substitution effects. A conclusion is given in the last section.

1.2. Overview on alternative real assets

Broadly speaking, asset classes contain investments with similar features including their risk and return characteristics and their sensitivity towards major market factors. The investment universe is usually divided into traditional asset classes and non-traditional or alternative asset classes that emerged over the last few decades. Traditional asset classes include equities, nominal government or corporate bonds. Examples for non-traditional or alternative asset classes are private equity or hedge fund investments.

The rationale for investing in the different asset classes varies broadly, which is in turn the reason why investors should diversify their asset allocation. For equity investments, a main investment objective is to participate in the growth potential and payout of listed companies. The rationale for bond investments includes income generation, deflation protection and reduction of portfolio volatility.

Traditional real assets include commodities, inflation-linked bonds, and real estate investments (direct or indirect, but not listed real estate investments). The categorization to this asset class subset is usually based on either the inflation protection feature such as real return, or the tangibility of the assets (see [3], p. 20, for a definition of real assets). Alternative real assets are represented by investments in shipping, aviation, infrastructure, or renewable energy facilities like photovoltaic or wind power plants. The real assets that provide the necessary cash flows may be roads, ships or solar plants. Therefore, alternative real assets provide direct exposure to specific projects and the corresponding "real facilities" but no exposure to a listed company's capital structure. Due to the project finance character, alternative real assets are not very liquid investments, even though secondary markets are already emerging. As revenues of the financed projects are often linked to inflation, most alternative real assets provide some inflation protection.

In the following we will focus on investments in solar power plants. This kind of alternative real assets is characterized by cash flows that are mainly driven by solar radiation and preferential feed-in tariffs. These cash flows are often linked to inflation and show a sensitivity to interest rates due to the partial debt financing of the photovoltaic projects. More details on the modeling of photovoltaic investments are given in the next section.

1.3. Modeling photovoltaic investments

1.3.1. *General approach*

When developing models for alternative real assets one might encounter problems due to the special design of this asset class. A top-down approach is usually oversimplifying. It is not appropriate to link alternative real asset returns to the returns of the equity market using the capital asset pricing model (CAPM), as we do not find listed companies that are comparable to these highly individual projects. However, this is the only approach taken in literature concerning the analysis of alternative real assets so far. For example [12] apply the CAPM approach to shipping investments.

For further comprehension a few words on the photovoltaic technique: photovoltaic is the direct generation of electricity out of global solar irradiance. This method of power generation is generally implemented through semiconductor panels, which are making use of a special case of the photoelectric effect. These solar panels are arranged in large arrays with additional infrastructure, so-called photovoltaic solar power plants. Using sunlight as the major resource, these facilities convert global irradiance into electricity in an environment-friendly way and are therefore categorized as renewable energy power plants.

As a matter of fact, the return structure of limited partnership funds financing photovoltaic projects is completely different from stock returns of solar sector companies (a company producing solar panels, for example). On the one hand, we have the direct exposure to a specific project with a well-defined revenue and cost structure, on the other hand we have exposure to a broad mix of risk factors that drive the stock returns of a listed company:

- capital structure of the company,
- balance sheet effects,
- general economic environment,
- change in exchange rates,
- differing expectations of market participants,
- analyst appraisals of profit expectations.

Therefore, we believe, that a different methodology than the top-down CAPM approach is required, when dealing with this kind of project finance. However, in relevant academic and practitioner publications, frequently top-down approaches are used for evaluating alternative real assets.

Based on expert information from a well known German initiator of photovoltaic projects, (we would like to thank KGAL (KG Allgemeine Leasing GmbH & Co.) for valuable insights to the financial modeling of solar plants.) we analyzed the revenue and cost structure reflecting the return profile of individual solar power plants. Thus, we were able to identify crucial and cash flow relevant aspects of photovoltaic projects that should be covered by an appropriate model:

- Revenues of the sale of electricity
- Cost of acquisition
- Operating expenses
- Interest on debt capital
- Tax payments and interest subsidy

Modeling these building blocks properly is at the heart of our bottom-up approach. [7] follow a similar bottom-up approach to derive the cash flows resulting from shipping investments. However they combine this procedure with a CAPM-approach to finally be able to price a shipping investment. Recently, at the stock market exchanges of Hamburg and Hannover a ship fund index is calculated using a bottom-up methodology. The index calculation is described in [14].

1.3.2. *Definition of the investment project*

As we do not focus on evaluating a highly specific photovoltaic project for a certain investor, we set up a typical photovoltaic project that disregards individual, location dependent features of an existing photovoltaic power station, and instead accentuates general properties of a typical project in a certain region. Solar power plants are usually set up in countries supporting renewable energy projects by an attractive compensation for feeding "clean" electricity into the grid. Such feed-in tariffs are guaranteed by local law and paid by the government, securing cash inflow. In some countries the compensation is even linked to local consumer price indices and accordingly provides partial inflation protection to potential investors.

Due to generous (inflation-linked) government compensations and a high level of global irradiance, Spain is a favored country for building up photovoltaic facilities and feeding the produced electricity into the local grid. For further comprehension, global irradiance is the total incoming solar radiation on a horizontal plane at the Earth's surface, i.e. the incoming solar radiation after scattering effects in the Earth's atmosphere. Therefore, our representative photovoltaic project is located in Spain and we have chosen a corresponding parametrization. This includes a project duration of 25 years which corresponds to the maximum duration of the attractive feed-in tariffs.

When following a bottom-up approach, it is absolutely necessary to model the project's properties and the corresponding dependencies (especially the dependencies on stochastic risk factors) with a high level of detail. To be able to model the cash flow structure in an appropriate way, we set up a comprehensive bottom-up framework covering all revenue and cost related features of the sample solar plant. An overview of the framework components and properties can be found in Table 1.1.

A special focus of our analysis is on the simulation of stochastic risk factors that drive the revenue and the cost side of the photovoltaic project. The key risk factors and their dependencies to various components of the bottom-up framework are listed in Table 1.2.

These risk factors have to be modeled and simulated in an integrated

Table 1.1. Comprehensive Framework for Bottom-up Modeling

Revenue	Modeling of the generation of electricity out of solar radiation, and the corresponding feed-in tariffs, including inflation-indexed tariff adjustments
Costs	Considering acquisition and operating costs, as well as inflation-driven increase over time
Financing	Modeling of equity, debt capital, and shareholder debt capital, as well as required liquidity reserves
Tax	Considering tax payments on company level, tax deductibility of interest payments on debt and shareholder debt, and tax loss carry-forwards to future periods
Facility	Considering annual power loss of the solar panels due to degradation effects, liquidation proceeds are on a par with deconstruction costs at project termination

Table 1.2. Dependencies of Bottom-up Framework on Key Risk Factors

Global Irradiance	$\longrightarrow$	Revenue
Inflation	$\longrightarrow$	Revenue, Costs
Nominal Interest	$\longrightarrow$	Financing

approach. This allows us to conduct a multi-scenario analysis of the inherent risk factors, the sample photovoltaic project and a portfolio consisting of traditional assets and the photovoltaic project.

1.3.3. *Modeling of risk factors*

1.3.3.1. *Economic factors*

The theoretical framework used for the analysis of alternative real assets in a portfolio context is the risklab Economic Scenario Generator (ESG).(Compare [16].) This model incorporates fundamental macroeconomic factors to describe the evolution of interest rates and equities. Using a cascade structure, it captures the long-term economic relationships while allowing for short-term deviations. This setting allows for an integrated modeling of financial markets, delivering economically meaningful and consistent scenarios.

We assume an arbitrage-free, frictionless financial market in continuous time $t \in [0, T^*]$, where the uncertainty in the market is described by the complete filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$. Details on this framework can be found in [17]. We use a non-defaultable money market account defined by $P(0,t) = \int_0^t \exp(r^N(s)ds)$ as numeraire, where the process $\{r^N(t)\}_{t\in[0,T^*]}$ is the nominal short rate. We assume the existence of a measure $\mathbb{Q}$ equivalent to $\mathbb{P}$, under which all discounted price processes of the financial market under consideration are martingales. The before mentioned cascade structure is now imposed on this foundation to incorporate long-term economic dependencies.
The model's first cascade comprises inflation $\{i(t)\}_{t\in[0,T^*]}$ and economic growth $\{w(t)\}_{t\in[0,T^*]}$, which are modeled by Vasicek processes, introduced in [15]. Under the equivalent martingale measure $\mathbb{Q}$ they are specified as follows:

$$di(t) = [\theta_i - \hat{a}_i \cdot i(t)]dt + \sigma_i dW_i^{\mathbb{Q}}(t),$$
$$dw(t) = [\theta_w - \hat{a}_w \cdot w(t)]dt + \sigma_w dW_w^{\mathbb{Q}}(t),$$

with the positive real numbers $\theta_i, \theta_w, \hat{a}_i, \hat{a}_w, \sigma_i, \sigma_w$ and the independent standard Brownian motions $W_i^{\mathbb{Q}}$ and $W_w^{\mathbb{Q}}$.

The second cascade contains the interest rate processes. The real short rate $\{r(t)\}_{t\in[0,T^*]}$ is modeled by a two factor Hull White model. Its dynamic is specified as

$$dr(t) = [\theta_r(t) + b_{rw} \cdot w(t) - \hat{a}_r \cdot r(t)]dt + \sigma_r dW_r^{\mathbb{Q}}(t),$$

with the positive real numbers b_{rw}, $\hat{a}_r$, and σ_r, the time dependent deterministic function θ_r and the standard Brownian motion $W_r^{\mathbb{Q}}$, independent of the Brownian motions mentioned before. The nominal short rate $\{r^N(t)\}_{t\in[0,T^*]}$ is defined as the sum of real short rate and inflation, i.e.

$$r^N(t) = r(t) + i(t).$$

The term structure of interest rates can then be derived by the zero-coupon bond prices obtained from this setting. As shown in [16], p. 4254f, the price of a zero-coupon bond with maturity $t < T \leq T^*$ is given by

$$P(t,T) = \exp(A(t,T) - B(t,T)r(t) - C(t,T)i(t) - D(t,T)w(t)),$$

where

$$\begin{aligned}
B(t,T) &= \tfrac{1}{\hat{a}_r}(1 - exp(-\hat{a}_r(T-t)),\\
C(t,T) &= \tfrac{1}{\hat{a}_i}(1 - exp(-\hat{a}_r(T-t)),\\
D(t,T) &= \tfrac{b_{rw}}{\hat{a}_r} \cdot \left(\tfrac{1-exp(-\hat{a}_r(T-t))}{\hat{a}_w} + \tfrac{exp(-\hat{a}_w(T-t))-exp(-\hat{a}_r(T-t))}{\hat{a}_w - \hat{a}_r}\right),\\
A(t,T) &= \textstyle\int_t^T (\tfrac{1}{2}(\sigma_r^2 B(l,T)^2 + \sigma_i^2 C(l,T)^2 + \sigma_w^2 D(l,T)^2) - \theta_r(l)B(l,T)\\
&\quad -\theta_i C(l,T) - \theta_w D(l,T))dl.
\end{aligned}$$

Using Girsanov's Theorem, the model equations can be derived under the real measure $\mathbb{P}$ instead of the equivalent martingale measure $\mathbb{Q}$ by replacing $W_i^{\mathbb{Q}}, W_w^{\mathbb{Q}}, W_r^{\mathbb{Q}}$ with the independent standard Brownian motions W_i, W_w, W_r and using the parameters $a_i = \hat{a}_i - \lambda_i \sigma_i^2$, $a_w = \hat{a}_w - \lambda_w \sigma_w^2$, and $a_r = \hat{a}_r - \lambda_r \sigma_r^2$. λ_i, λ_w, and λ_r are obtained by the change of measure, as shown e.g in [10]. As can be seen, the term structure of interest rates is driven by the real short rate process, as well as the underlying macroeconomic factors of economic growth and inflation.

The third cascade contains equity assets. The equity prices $\{S_t^E\}_{t\in[0,T^*]}$ are driven by the following dynamics:

$$dS^E(t) = [\alpha_E + b_{Er}r(t) - b_{Ei}i(t) + b_{Ew}w(t)]\, S^E(t)dt + \sigma_E S^E(t)dW_E(t),$$

where b_{Er}, b_{Ei}, b_{Ew}, and σ_E are positive real numbers, $\alpha_E \in \mathbb{R}$ and $W_E(t)$ is a standard Brownian motion, independent of those mentioned above.

1.3.3.2. *Non-economic factors*

The final model component needed, which will add to the basic framework described above, is yearly global irradiance on a horizontal surface $\{Y(t)\}_{t\in\{0,1,\ldots,T^*\}}$, which is the starting point for the generation of electricity in a solar plant. An appropriate starting point for the modeling approach is average monthly global irradiance. Having a higher frequency will, apart from data availability issues, increase the noise component of the model, but not have a different impact on at most monthly simulated cash flows generated by the photovoltaic plant. As the irradiance model component independently adds to the basic framework, and the frequency of our data is also monthly, we develop a discrete-time model. The following sections now present the building blocks for the derivation of a discrete-time model for yearly global irradiance.

Historical analysis of monthly global irradiance

To model monthly global irradiance on a horizontal surface, $\{Y^M(m)\}_{m\in\{1,...,12\cdot T^*\}}$, where m indicates the month, we use a simple Angström-type equation, where bright sunshine duration is related to global irradiance. An overview of Angström-type equations to model global irradiance on horizontal surfaces is given in [1]. We follow a parsimonious approach, where we use a sinusoidal function to describe monthly global irradiance. This offers the advantage of not requiring information on bright sunshine hours, day length, and extraterrestrial radiation. We specifically assume monthly global irradiance to follow the process

$$Y^M(m) = A + B\sin(\frac{2\pi}{12}m^* - F) + \epsilon(m), \tag{1.1}$$

where A is the average monthly global irradiance, B is the amplitude of seasonal radiation variation, F is the phase angle in radian measure, i.e. the necessary shift of the function on the time axis, and $m^* \in \{1, 2, ..., 12\}$ is the number of a specific month in the year, i.e. the number of January is 1, etc. (Compare [2].) The zero-mean process $\{\epsilon(m)\}_{m\in\{1,...,12\cdot T^*\}}$ represents random fluctuations around the seasonal trend of monthly global irradiance. We use a least squares method to estimate the parameters A, B, and F in (1.1), and conduct the estimation on annualized monthly global irradiance data ranging over 12 years from 1/1982 to 12/1993 for several locations in Spain. (For nonlinear least squares estimation, see e.g. [9].) Our data source is the online archive of the U.S. Department of Energy's National Renewable Energy Laboratory and the World Radiation Data Center (`http://wrdc-mgo.nrel.gov/`). The estimated model produces a high degree of variance explanation, with $R^2 > 0.9$ for all locations. R^2 is calculated as $1 - \frac{Var(\hat{\epsilon}_m)}{Var(\hat{Y}(m))}$, where $\hat{Y}(m)$ denotes the observed radiation series. Exemplarily for the location Toledo, the fitted series and the original series can be seen in Figure 1.1. The fitted series is close to the original series with the model explaining 98% of the variation in the original series. The analysis of the model residual time series $\{\hat{\epsilon}_m\}_{m\in\{1,...,12\cdot T^*\}}$ shows skewed distributions. The estimated density of residuals for the location Toledo is shown in Figure 1.1. The density was estimated using Gaussian kernel smoothing, i.e. the density estimator

$$\hat{f}(Y) = \frac{1}{Th}\sum_{t=1}^{T}(\frac{1}{\sqrt{2\pi}}e^{-\frac{(Y-\hat{Y}(t))^2}{2h^2}})$$

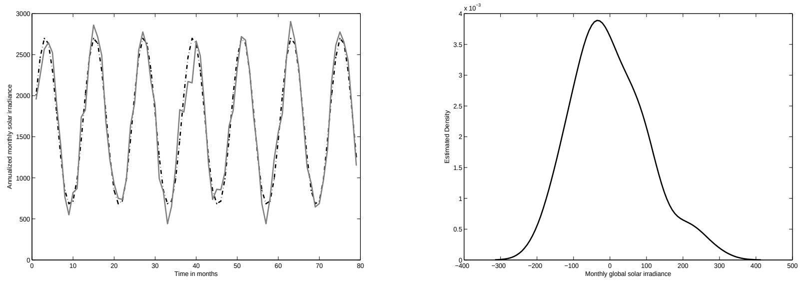

Fig. 1.1. Left: Original (continuous line) and fitted (dashed line) annualized monthly global radiation series. Right: Estimated density of residuals

was applied to the observed data series, where the bandwidth used was $h = (\frac{4}{3T})^{\frac{1}{5}}\sigma$, and σ was obtained using the median absolute deviation estimator. (More information on kernel smoothing can be found e.g. in [4]). To capture the skewness in the fluctuation around the average seasonal global irradiance, we specify the noise process $\{\epsilon_m\}_{m\in\{1,\ldots,12\cdot T^*\}}$ to follow a Markov switching model, i.e.

$$\epsilon(m) = \mu_1^M + X^M(m)(\mu_2^M - \mu_1^M) + \\ + \left(\sigma_1^M + X^M(m)(\sigma_2^M - \sigma_1^M)\right) \cdot u^M(m), \quad (1.2)$$

where μ_1^M, $\mu_2^M \in \mathbb{R}$, σ_1^M,$\sigma_2^M \in \mathbb{R}^+$,$\{X^M(m) : X^M(m) \in \{0,1\}\}_{m\in\{1,\ldots,12\cdot T^*\}}$ is a time-homogenous Markov chain and $u^M(m)$ is the increment of a standard Brownian motion, independent of $X^M(m)$. Details on Markov Switching Models can be found e.g. in [13]. To obtain parameter estimates we are applying a maximum likelihood approach to the residual time series.(For the maximum likelihood estimation of Markov switching models, see e.g. [8].) The residual distribution from the estimated model, obtained by a Monte carlo simulation, is shown for Toledo in Figure 1.2. We conduct a two-sample Kolmogorov-Smirnov test on the simulated and the original monthly global irradiance series, which does not show evidence against both series coming from the same distribution. (For the two-sample Kolmogorov-Smirnov test, see e.g. [6].)

The out-of-sample performance of the estimated model can be seen in Figure 1.3, exemplarily for Toledo. The original time series are shown along with the 1%, 50%, and 99% quantiles of a simulation of 1000 paths following (1.1) and (1.2).

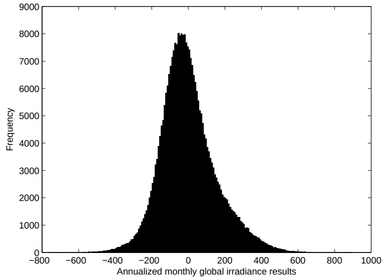

Fig. 1.2. Histogram of simulated monthly global irradiance residuals

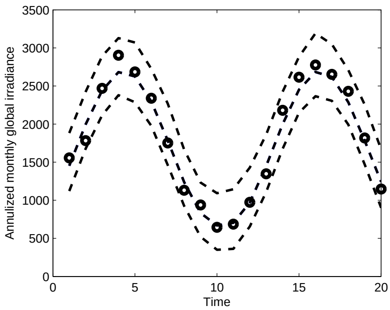

Fig. 1.3. Original series (dots) and annualized monthly global irradiance quantiles (barred lines)

Monte carlo analysis of yearly global irradiance

We use the results for monthly global irradiance to analyze yearly global irradiance, which is needed for the analysis of the photovoltaic investment. The global irradiance of a specific year is just the sum of the monthly global irradiance over that year. Formally, the global irradiance of year t, $Y(t)$, with $t \in \{\frac{m}{12} : m \in \{1, ..., 12 \cdot T^*\} \wedge m \mod 12 = 0\}$ is given by

$$Y(t) = \sum_{j=1}^{12} Y^M(12t - j + 1). \tag{1.3}$$

We now use the model for $Y^M(m)$ to obtain the distribution of $Y(t)$ by simulating 1000 trajectories of monthly global irradiance, each over a time span of 25 years. By calculating yearly radiation on the simulated processes according to (1.3), we generate 1000 paths of yearly global irradiance with 25 observations per path. The resulting distribution can be seen in Figure 1.4 for Toledo. Compared to monthly global irradiance the skewness is re-

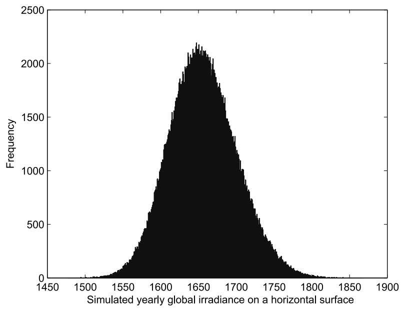

Fig. 1.4. Histogram of simulated yearly global irradiance

duced, but retained. The results we obtain for yearly global irradiance are similar to the published information on average yearly global irradiance, as available for example through radiation maps shown in [11] or offered on the website `www.meteonorm.com`, which provides a comprehensive meteorological reference. Table 1.3 below shows the means and relative standard deviations, i.e. the standard deviation divided by the mean, resulting from the simulation of yearly global irradiance for several sites, as well as the R^2-values of the estimated model. Comparing means and standard devia-

Table 1.3. Yearly global irradiance statistics for various locations in spain

Location	Mean in KWh/m^2	Rel. Standard Deviation	R^2
Toledo	1655.96	2.67%	95.9%
Caceres	1661.07	2.86%	95.3%
Logrono	1407.48	3.06%	95.5%
Madrid	1602.93	3.05%	96.1%
Murcia	1698.89	2.00%	96.9%
Mallorca	1556.73	1.92%	97.7%
Santander	1205.08	3.12%	94.9%
Sevilla	1658.85	1.95%	97.2%

tions with the information in [11] shows them to be in line with their data. The skewness of the distributions for the different locations is very similar, the distributions vary mainly in location and scale. For the following analysis of photovoltaic investments, we choose Toledo as a reference for yearly global irradiance on a horizontal surface, because it shows average characteristics of global irradiance in Spain, and is located close to a large number of existing solar plants.

1.4. Photovoltaic investments in a portfolio context

1.4.1. *Setting the portfolio context*

We analyze the impact of photovoltaic investments in a portfolio context by evaluating changes in risk/return profiles. Apart from photovoltaic investments, the asset class universe underlying this analysis comprises equity and bonds, represented by a bond index, which is constructed by periodically buying and selling bonds such that a constant modified duration of approximately 6 years is achieved. (The necessary bond prices are determined by the stochastic evolution of interest rate term structures generated using the integrated economic model.) The specific characteristics of these asset classes in terms of risk (measured by the standard deviation of annual returns) and expected return (measured by the mean of annual returns) over the considered investment horizon of 25 years, are given in the following Table 1.4.

Table 1.4. Characteristics of traditional asset classes

	Return p.a.	Risk p.a.
Equity	7.7%	14.8%
Bonds	4.3%	3.5%

Three types of investors are characterized by three different initial asset allocations. These allocations, presented in Table 1.5, are the starting point for our analysis.

The three allocations represent different risk (and return) preferences with 10%, 20% and 30% equity allocations. Comparing the risk and return figures for the "10% Equity" portfolio in Table 1.5 to the pure bond profile in Table 1.4, we can already see the benefits of diversification, i.e. an increase in expected return at the same level of expected risk when a 10% allocation

Table 1.5. Initial asset allocations

Initial Allocation	Equity	Bonds	Return p.a.	Risk p.a.
"10% Equity"	10%	90%	4.9%	3.5%
"20% Equity"	20%	80%	5.3%	4.1%
"30% Equity"	30%	70%	5.6%	5.1%

in bonds is substituted by equities.

Based on the three initial portfolios in Table 1.5, we evaluate the changes in risk and expected return figures when adding a maximum of 5%, 15%, or 25%, and up to 100% of photovoltaic investments.

1.4.2. *Including photovoltaic investments in a portfolio*

As a mark-to-market modeling of a photovoltaic investment is impossible without very strong assumptions (e.g. regarding the appropriate discount rate), we make use of an approach that does not require a pricing of the portfolio at each point in time. Furthermore, we attached importance to a realistic modeling of the investment itself. Starting with a predefined initial portfolio consisting of the traditional assets, i.e. equities and bonds, a portion of an investment in a photovoltaic project is added. Incoming cash flows from the photovoltaic project are reinvested into the traditional part of the portfolio. In each time step, the reinvestment is allocated according to the weights of the traditional assets in the portfolio. As a result of this approach, the absolute allocations of the photovoltaic investment decrease over time.

A reinvestment of the dividends from the photovoltaic investment in other (new) projects, in order to maintain the relative photovoltaic investment, would imply arbitrary availability and divisibility. An investor would not enter into new photovoltaic projects on an (semi-)annual basis as dividends flow back.

We perform the portfolio analysis on log-returns, which offers, for example, the advantage that multi-period log-returns can be calculated as the sum of the single-period log-returns. Due to the before-mentioned mark-to-market problems of photovoltaic investments, we cannot calculate portfolio returns before the end of the investment horizon. Using log-returns allows us to easily transform observable total return characteristics over a period of several years to more common per annum values. Consider therefore portfolio p, which comprises photovoltaic investments. Thus, the portfolio value, and therewith the single-period returns cannot be calculated during the investment horizon, as the value of the photovoltaic investment is unknown during the investment horizon. Assume nevertheless, that the (unobservable) single-period log-returns $\{r_{p,i}\}$ for the periods $i = 1, ..., n$, satisfy $Var(r_{p,i}) = \sigma_p^2 \; \forall \; i = 1, ..., n$ and $Cov\,(r_{p,i}, r_{p,i-j}) = k_p(j) \cdot \sigma_p^2$ for $j = \pm 1, \pm 2, ...$. The multi-period or total log-return $r^{total}_{p,1\mapsto n}$ is obtained by summing up the single-period log-returns, i.e.

$$r^{total}_{p,1\mapsto n} = \sum_{i=1}^{n} r_{p,i}.$$

Given the assumptions, it is easy to show that (for this result compare also e.g. [5], p. 49.)

$$Var(r^{total}_{p,1\mapsto n}) = n \cdot (1 + 2\sum_{j=1}^{n-1} (1 - \tfrac{j}{n}) \cdot k_p(j)) \cdot \sigma_p^2. \qquad (1.4)$$

With

$$\begin{aligned} Var(r^{total}_{p,1\mapsto n}) &= \sum_{i=1}^{n} Var(r_{p,i}) + \sum_{i\neq j} Cov\,(r_{p,i}, r_{p,j}) \\ &= \sum_{i=1}^{n} Var(r_{p,i}) + 2 \cdot \sum_{j=1}^{i-1} Cov\,(r_{p,i}, r_{p,j}) \\ &= \sum_{i=1}^{n} (\sigma_p^2 + 2 \cdot \sum_{j=1}^{i-1} (\sigma_p^2 \cdot k_p(i-j))) \\ &= \sigma_p^2 (n + 2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{i-1} k_p(i-j)). \end{aligned}$$

Now, defining $k := i - j$, we have

$$\begin{aligned} \sum_{i=1}^{n} \sum_{j=1}^{i-1} k_p(i-j) &= \sum_{i=1}^{n} \sum_{k=1}^{i-1} k_p(k) \\ &= \sum_{k=1}^{n-1} \sum_{i=k+1}^{n} k_p(k) \\ &= \sum_{k=1}^{n-1} (n-k) \cdot k_p(k), \end{aligned}$$

where we used the fact that $1 \leq i \leq n, 1 \leq k \leq i-1 \iff 1 \leq k \leq n-1, k+1 \leq i \leq n$ in the second equality. Factoring out n, proposition (1.4) follows. Using this result, we define a volatility adjustment factor

$$\alpha := (1 + 2\sum_{j=1}^{n-1} (1 - \frac{j}{n}) \cdot k_p(j)). \qquad (1.5)$$

Thus, knowing the degree of autocorrelation and having for instance yearly periods, allows us to calculate the annualized volatility from total volatility by dividing the total volatility by the square root of the number of years adjusted for the degree of autocorrelation by the factor $\frac{1}{\alpha}$. Assuming the absence of autocorrelation, only the observable initial portfolio value and the observable portfolio value at the end of the investment horizon are needed to calculate single-period portfolio log-return volatility. The dependence of cash flows generated by a solar power plant over time should mainly be driven by their dependence on inflation and interest rates, which both have a negative impact on the cash flows, and both show positive autocorrelation. Thus, one would expect positive autocorrelation, if any, in the single-period returns of a photovoltaic investment. Positive autocorrelation is increasing α defined in (1.5), and therefore would increase the measured total variance of a portfolio including photovoltaic investments. Scaling the total variance by the number of periods will then overstate the single-period variance, which makes our assumption of absence of autocorrelation and therewith the results of our analysis conservative.

1.4.3. *Results*

Tables 1.6 to 1.8 present the results of our portfolio optimizations for the three initial allocations "10% Equity", "20% Equity" and "30% Equity". Every table shows the resulting optimized portfolios for similar expected risk or similar expected return statistics when we allow the allocation of a maximum weight of 5%, 15% or 25% to the photovoltaic investment opportunity. The "higher return" or "lower risk" portfolios, denoted by "HR X% PV", and "LR X% PV", respectively, are given in the three tables together with the corresponding expected risk and expected return figures. (X denotes the photovoltaic investment proportion in the portfolio.) These efficient portfolios (with weights for photovoltaic (PV), equity (E) and bond (B) investments), compared to the initial allocation, yield the same expected return with less risk, or the same risk with more expected return. The last three columns of Tables 1.6 to 1.8 present the weights differences between the initial allocations and the new efficient weights, i.e. the substitutional effects due to the introduction of photovoltaic investments.

The dominance of portfolios including the photovoltaic investment opportunity is striking. In Table 1.6 we see the results for an initial equity allocation of 10%. The higher return portfolios show an increase in expected return from 20bp up to 70bp depending on the maximum quota of photovoltaic investments in the portfolio. For example, the higher return portfolio with a limit of 5% in photovoltaics ("HR 5% PV") allocates the maximum of 5% to photovoltaic investments, 13% to equities and 82% to bonds, increasing the expected return from 4.9% to 5.1% at the same level of expected risk (3.5%). The lower risk portfolios ("LR X% PV") decrease the expected risk by up to 60bp (or approximately 17% in relative terms) at the same level of expected return. With higher photovoltaic limits, the risk

Table 1.6. Results for the initial allocation "10% Equity"

	Allocations			**Statistics**		**Substitution w.r.t. "10% Equity"**		
Portfolio	**PV**	**E**	**B**	**Return**	**Risk**	**PV**	**E**	**B**
"10% Equity"	**0%**	**10%**	**90%**	**4.9%**	**3.5%**	-	-	-
"HR 5% PV"	5%	13%	82%	5.1%	3.5%	+5%	+3%	-8%
"HR 15% PV"	15%	15%	70%	5.4%	3.5%	+15%	+5%	-20%
"HR 25% PV"	25%	15%	60%	5.6%	3.5%	+25%	+5%	-30%
"LR 5% PV"	5%	7%	88%	4.9%	3.3%	+5%	-3%	-2%
"LR 15% PV"	15%	5%	80%	5.0%	3.1%	+15%	-5%	-10%
"LR 25% PV"	25%	4%	71%	5.2%	2.9%	+25%	-6%	-19%

Table 1.7. Results for the initial allocation "20% Equity"

	Allocations			**Statistics**		**Substitution w.r.t. "20% Equity"**		
Portfolio	**PV**	**E**	**B**	**Return**	**Risk**	**PV**	**E**	**B**
"20% Equity"	**0%**	**20%**	**80%**	**5.3%**	**4.1%**	-	-	-
"HR 5% PV"	5%	21%	74%	5.4%	4.1%	+5%	+1%	-6%
"HR 15% PV"	15%	21%	64%	5.7%	4.1%	+15%	+1%	-16%
"HR 25% PV"	25%	21%	54%	5.9%	4.1%	+25%	+1%	-26%
"LR 5% PV"	5%	17%	78%	5.3%	3.8%	+5%	-3%	-2%
"LR 15% PV"	15%	12%	73%	5.3%	3.3%	+15%	-8%	-7%
"LR 25% PV"	25%	7%	68%	5.3%	3.0%	+25%	-13%	-12%

and return characteristics cannot even be brought down to the initial levels when selecting portfolios from the efficient set. The minimum expected return for "LR 15% PV" is 5.1%, for "LR 25% PV" it is 25%, respectively.

In Table 1.7 we see higher return and lower risk portfolios for an initial allocation of 20% in equites. The improvement on the return side ranges from 10bp to 60bp and up to 110bp (or approximately 27% in relative terms) on the risk side. The comparison of the optimized portfolios to a

Table 1.8. Results for the initial allocation "30% Equity"

	Allocations			**Statistics**		**Substitution w.r.t. "30% Equity"**		
Portfolio	**PV**	**E**	**B**	**Return**	**Risk**	**PV**	**E**	**B**
"30% Equity"	**0%**	**30%**	**70%**	**5.6%**	**5.1%**	-	-	-
"HR 5% PV"	5%	30%	65%	5.8%	5.1%	+5%	0%	-5%
"HR 15% PV"	15%	29%	56%	6.0%	5.1%	+15%	-1%	-14%
"HR 25% PV"	25%	28%	47%	6.2%	5.1%	+25%	-2%	-23%
"LR 5% PV"	5%	27%	68%	5.6%	4.7%	+5%	-3%	-2%
"LR 15% PV"	15%	21%	64%	5.6%	4.1%	+15%	-9%	-6%
"LR 25% PV"	25%	15%	60%	5.6%	3.5%	+25%	-15%	-10%

30% equity portfolio in Table 1.8 reveals up to 60bp more return and up to 160bp (or approximately 31% in relative terms) less risk.

Interestingly, the optimized portfolio allocations in the case of the "10% Equity" and "20% Equity" show a slight increase in the equity weight for the higher return portfolios, as the photovoltaic investment seems to reduce the portfolio risk substantially. Therefore, the bond allocation is reduced by up to 30% ("HR 25% PV" with "10% Equity" as initial allocation). For the "30% Equity" portfolio the optimized higher return portfolios show a stable allocation in equities - bonds are substituted for by the allocation to photovoltaic investments. When we optimize the lower risk portfolios the allocation to photovoltaic investments consistently substitutes equity and bonds. For the "20% Equity" portfolio both substituted at similar amounts, for the "30% Equity" portfolio compared to bonds more equity is substituted, and for "10% Equity" portfolio more bonds are substituted compared to equity.

1.5. Conclusion

Investing in photovoltaic facilities is environmentally supportive. In addition photovoltaic investments are an attractive asset class from an investment perspective. In our analysis, the maximum weight allowed is allocated in all of our optimized portfolios. We see a straightforward tendency that, including photovoltaic investments in a portfolio, decreases risk and/or increases expected return.

To derive these results we make use of a comprehensive framework to analyze the attractiveness of photovoltaic investments in a multi-asset portfolio. Based on our bottom-up approach, we are able to identify the quantitative characteristics of photovoltaic investments in a portfolio context.

We optimized portfolios including photovoltaics with respect to initial allocations of 10%, 20% and 30% in equities, respectively. These base allocations reflect different risk and return preferences of representative investors. Depending on the preferences and the maximum allowed photovoltaic weight, optimized higher return portfolios provide increases in expected return of up to 70bp while the lower risk portfolios decrease the risk figures by up to 1.6% in absolute or by 31% in relative terms.

As a result, we find that in optimized higher return portfolios photovoltaic investments substitute bonds - allowing slight increases in the equity quota at the same time. For the lower risk portfolios, the allocation to photovoltaic investments is financed by decreasing bond and equity weights at the same time. The reduction of bond and equity allocations is approximately the same.

The chosen bottom-up framework allows for the inclusion of alternative real assets when evaluating and optimizing portfolio allocations. We can examine the portfolio contribution of alternative real assets in general and we are able to customize the modeling for specific investment assets in great detail. Beside the analysis in a portfolio context, this bottom-up modeling also allows ongoing monitoring of specific alternative real assets.

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