Index tracking UNDER TRANSACTION COSTS:

rebalancing passive portfolios

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Portfolio managers must be able to estimate transaction costs when constructing and rebalancing portfolios. This is not only true for active but also for passive investment styles. Index trackers are especially vulnerable, as they are allowed only very small deviations from their benchmarks. The following article demonstrates how transaction costs can be estimated and incorporated in the portfolio construction process. First, we develop the RiskLab Transaction Cost Model (TraC'M). In a second step we include transaction costs in a standard index tracking model. Finally, we show how portfolio managers can improve their overall performance when regarding transaction costs.

1. STATEMENT OF THE PROBLEM

The performance of a portfolio is determined not only by the quality of the investment strategy but also by the terms of execution. The potential outperformance of an active strategy can be heavily imparted if the prices of stocks that are selected for inclusion in the portfolio rise systematically between investment decision and complete trade execution. Especially for illiquid stocks this effect can be rather strong.

Similar effects are relevant for passive portfolio management, although this is often neglected. The stocks in a benchmark can usually not be traded at benchmark prices. This is one of the reasons why index 'tracking' is often preferred to exact replication. Various optimisation algorithms can be used to reproduce the performance of a benchmark while avoiding illiquid stocks with a high potential market impact.

The gains of reduced transaction costs, however, must be contrasted with a loss in replication exactness, which is inevitable. A suitable index tracking procedure must therefore incorporate an answer to the following problems:

- modelling of explicit and implicit transaction costs; and

- modelling of the trade-off between replication exactness and reduction of transaction costs.

The remainder of this article will show how these two problems can be solved and implemented in an index tracking environment. Section two will introduce a model for transaction cost estimation. Section three shows how this transaction cost model can be included in a portfolio construction process and section four concludes with a case study.

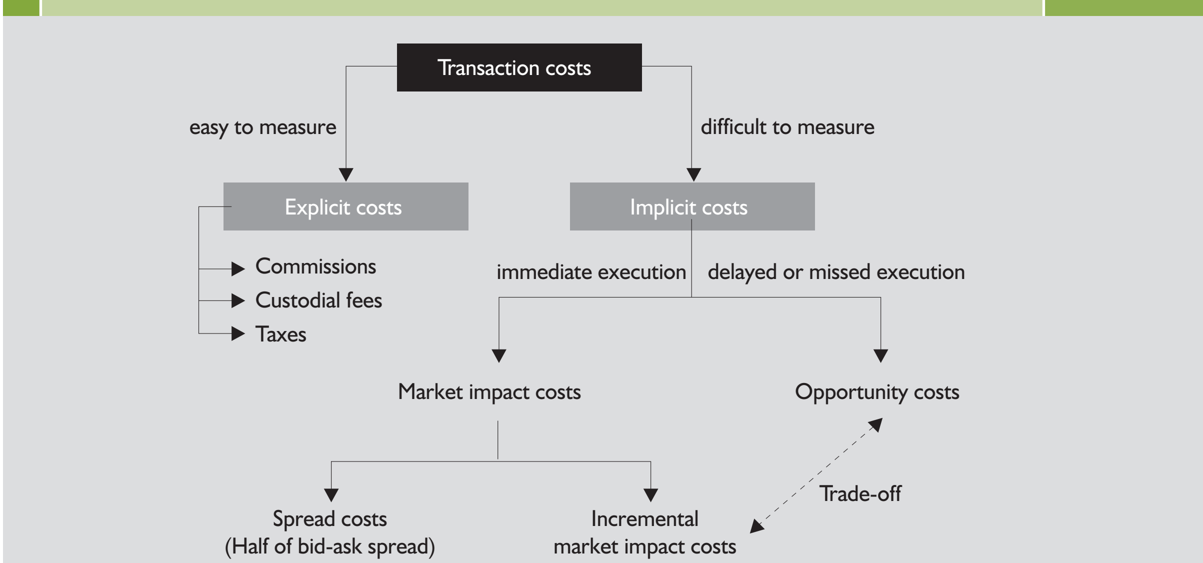
2. MODELLING TRANSACTION COSTS - THE RISKLAB TRANSACTION COST MODEL (TRAC'M)

In this section, we give a brief summary of the RiskLab Transaction Cost Model (TraC'M).[1] The TraC'M provides a forecast of the total transaction costs of an equity trade. According to [Kei98] among others, we decompose the total transaction costs *TC* into explicit costs *EC* and implicit costs *IC*. Explicit costs (also: direct costs, processing costs) are the direct costs of trading, such as broker commissions, custodial fees, taxes etc. Implicit costs (also: indirect costs) comprise market impact costs *MC* and opportunity costs *OC*.

Market impact cost is defined as the deviation of the transaction price from the 'unperturbed price' that would have prevailed if the trade had not occurred. In other words, market impact cost is the price an investor pays for immediate execution. Market impact is difficult to measure because the unperturbed price is not observable. In the TraC'M we choose the mid price of the quote just prior to the transaction as a proxy for the unperturbed price. Using this estimate for the unperturbed price, we further decompose the market impact cost *MC* into half the bid-ask spread *SP* and the incremental market impact cost MC^I.

The total bid-ask spread *SP* is the loss from buying one share of stock (at the ask) and immediately selling it (at the bid). Incremental market impact cost MC^I is the change in the stock price that occurs when the number of

Decomposition of total transaction costs **Exhibit 1**



Source: RiskLab GmbH

stocks an investors desires to buy or sell exceeds the number other market participants (e.g. market-makers) are willing to sell at the quoted ask or buy at the quoted bid. The second component of implicit cost is opportunity cost OC associated with missed trading opportunities. The notion of opportunity cost assumes that the trade is motivated by information that has decaying value over time. To capture this value, immediate execution is necessary. Putting it altogether, we define the total transaction costs (in costs per share) as

$$TC = EC + \frac{SP}{2} + MC^I + OC \qquad (1)$$

Exhibit 1 summarises the cost components and their relationships. In equation (1), explicit costs *EC* and spread costs *SP* are either directly observable or can be easily estimated. On the other hand, incremental market impact costs and opportunity costs are typically unobservable and difficult to estimate. Therefore, the focus of the TraC'M is on the modelling of the latter two cost components.

The main assumption underlying the TraC'M is a relation between incremental market impact costs and opportunity costs, which we call the reflection principle. Given an order execution time ≤ (in trading hours) , $\tau,\ 0 \le \tau \le \tau_{\text{No Impact}}$, it states that the following relation holds:

$$MC^I(\tau) = OC(\tau_{\text{No Impact}}) - OC(\tau) \qquad (2)$$

Here $MC^I(\tau)$ and $OC(\tau)$ denote the incremental market impact costs and opportunity costs, respectively, that accrue for an order executed within τ hours. The minimum time needed to execute an order without incurring any incremental market impact cost is denoted by $\tau_{\text{No Impact}}$. Given the opportunity cost function $OC(\tau)$, the incremental market impact function follows directly from (2). The economic reasoning behind the reflection principle is as follows: Opportunity costs tend to increase as the time between the decision to trade and the completion of the trade increases. On the other hand, incremental market impact cost typically decreases during this time interval, because a trader with more time can split up the transaction into smaller transactions that individually exert little or no price pressure.

Suppose a liquidity demander, e.g. a portfolio manager, wishes the immediate execution of a large order. To be executed, a liquidity provider, e.g. a market-maker, has to take positions in stocks and size he does not want to take. While clearing out these positions he faces two basic types of risk: inventory risk and adverse selection risk (see, e.g., [Hua94]). Inventory risk can be further split up into time-to-clear risk and asset price risk. The former refers to the unknown time to clear the stock's inventory and the latter to the unknown price for which the stock's inventory can be cleared. Adverse selection risk occurs because of information asymmetries between the trade parties. For taking these risks, the liquidity provider charges a return, the incremental market impact. Risk and return are linked by the market price of liquidity risk λ. Neglecting the adverse selection risk, the incremental market impact function in the TraC'M is modelled as:

$$MC^I(\tau) = \lambda \times f\,(\text{asset price risk, time-to-clear risk}), \qquad (3)$$

where f is a suitably chosen function. The time-to-clear risk is basically determined by three factors: the trade volume, the distribution of the trade volume over time and a parameter which represents the portion of the (daily) trade volume that can be traded without generating any impact. Asset price risk, on the other hand, is measured in terms of volatility.

3. THE REBALANCING PROBLEM UNDER MARKET IMPACT

Let us consider the situation of a portfolio manager responsible for a passive portfolio. We assume that at the current point in time the portfolio consists of holdings of a certain number of shares for each stock, or equivalently, we use the current portfolio weighting x_C. The current weighting of the original benchmark is denoted by x_B. If the portfolio manager is urged for some reason to adjust the current portfolio holdings (i.e. caused by a change in the index composition), he has to trade a certain volume V of money. This causes explicit and implicit transaction costs, where explicit costs are usually small in magnitude. If we assume that the portfolio manager wants to trade quickly, the main cost component we have to control for is market impact cost. The main question the portfolio manager has to answer then is the following:

How should the weights of the traded portfolio (denoted as x_T) be chosen to assure that the new portfolio x_N (current portfolio plus traded portfolio) is a good tracking portfolio for the current benchmark while the loss in the market impact occurring on portfolio x_T is small?

Problem setting

Using the definitions from the previous section, we can formulate the rebalancing problem as follows:

x_B vector of weights of the benchmark portfolio (e.g. EuroStoxx50)

x_C vector of current weights of the already existing tracking portfolio

x_T vector of weights of the portfolio to be traded

x_N vector giving the weights of the new portfolio after the trade, $x_N = x_C + x_T$

X_{admis} set of potential and admissible portfolios

$TE(x_N, x_B)$ tracking error of new portfolio with respect to the benchmark portfolio

$MC(\tau(x_T))$ market impact generated by trading portfolio x_T

U_{TE} upper bound on the tracking error

$$\begin{aligned}
&\min MC(\tau(x_T)) \\
&\text{subject to} \\
&x_N \in X_{admis} \\
&TE(x_N, x_B) \leq U_{TE}
\end{aligned}$$

For our specific test implementation we have used the following feasible sets and constraints:

$$TE(x_N, x_B) = (x_N - x_B)^T C (x_N - x_B),$$

$$X_{admis} = \{(x_1, \ldots, x_n) \mid 0 \leq x_i \leq 0.10, i = 1, \ldots, n\},$$

where C is the covariance matrix of asset returns. After some careful reformulation, the problem can be stated as a bilinear optimisation problem (i.e. with bilinear objective function) under convex constraints.

Some remarks on the choice of the optimisation environment

Usually, index tracking problems under various constraints are solved either by sophisticated optimisation algorithms (as provided by GAMS, AMPL, Matlab, etc) or by heuristic optimisation methods, as for example greedy algorithm, simulated annealing or genetic algorithm (see, e.g., [Der01] or [Gil02]). On the one hand, these heuristic methods are very well suited for complicated optimisation problems, especially including hard constraints (like non-convex or mixed-integer constraints) and further, they are fast and easily adapted to different instances of new portfolio problems. An appropriate environment is for example given by the decision support system as described in [Der01]. On the other hand, the methods are in general not real-time algorithms as required for our given problem. Hence, we only have the option to use superlinearly convergent sophisticated optimisation

Tracking error versus market impact — **Exhibit 2**

Tracking error	100	50	20	10	7.5	5	3.0	1.0	0.0
Market impact	26.4	26.4	26.4	28.6	33.7	40.8	48.2	57.4	62.3

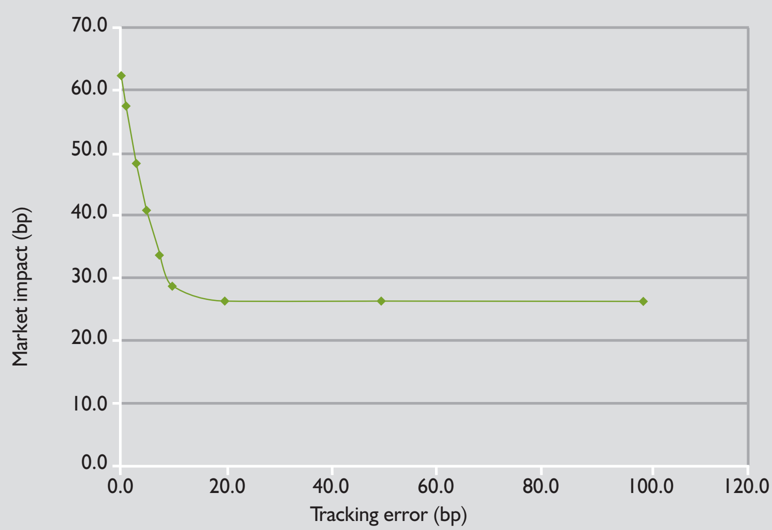
Source: RiskLab GmbH

routines, e.g. MINOS[2] provided with GAMS (cf. [GAM20]). Further, in this environment we can take advantage of the bilinear structure of the problem and thus are able to solve it efficiently. Of course, these algorithms are in general not able to yield the global optimum of the above bilinear rebalancing problem. Therefore we use a bi-level reformulation as final problem formulation for the numerical solution.

Numerical results of the portfolio problem

To illustrate the dependence of tracking error and market impact, we have computed an example based on data of the EuroStoxx50 with prices as of January 2, 2001. We have fixed a portfolio volume of €50m and we have estimated the necessary market impact parameters from historical data. In a first step, we compute the tracking error optimal portfolio, while neglecting market impact. If we neglect all additional constraints, the answer is trivial: The tracking portfolio coincides with the benchmark portfolio. Thus we are given a lower bound on the tracking error (in this case 0 basis points), together with an upper bound on the market impact of 62.3 basis

Trade-off between market impact and tracking error — **Exhibit 3**



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points. If we vary the tracking error from 0 to 100 basis points, we can compute Exhibit 2. The typical trade-off relationship between market impact and tracking error can be better illustrated in Exhibit 3.

Now it is up to the portfolio manager to choose one point from this frontier, i.e. he has to decide about his personal view on the optimal trade-off between tracking error and market impact.

Exhibit 3 reveals some further interesting facts: First, we see that there exists a portfolio with a market impact of 26.4 basis points and a tracking error of 20 basis points. Even if we allow larger tracking errors it is not possible to decrease market impact any more. In other words, the current state of the market does not allow to trade any portfolio with a volume of €50m with a market impact below 26.4 basis points.

4. CASE STUDY

Let us illustrate the discussion above with an example. Imagine that you want to set up a portfolio with a net asset value of €50m. You are supposed to track the EuroStoxx50, but the investor allows you a leeway of around 100bp per annum as he wants to keep his market impact small. Given his constraints you can suggest the following alternative strategies:

Strategy A: Fair tracking, small impact

You can think of the 100bp as a *tracking error budget* which can be used to keep your market impact small. One strategy is to spend all of this budget, i.e. in Exhibit 3 you would choose the point on the graph that corresponds to an annual tracking error of 100bp. Now you know that you can expect a market impact of 26.4bp when you set up the portfolio.

As described in section three, an optimiser can be used to calculate the respective portfolio holdings. The results are illustrated in Exhibits 4 and 5. Exhibit 4 shows a histogram of the active weights in the tracking portfolio. There are two underweighted stocks with –2.5% and –2% respectively. These underweights correspond to stocks with a large market impact. The optimisation compensates for these underweights by slightly overweighting numerous stocks by 0.5% and 1%. Only one stock has a larger overweight (2%). Exhibit 5 shows the history of daily differences between benchmark and portfolio performance for the following twelve months. The daily performance differences can be as large as 20 basis points, but there is no systematic cumulative out- or underperformance. The actual ex-post tracking error is around 120 basis points and exceeds the budget that has entered the optimisation. The difference between ex-ante and ex-post values is a common problem. There are two ways to account for this. First, one may be more restrictive in the optimisation and start with a value of, e.g., 80 instead of 100 basis points. Second, one may rebalance the portfolio from time to time.

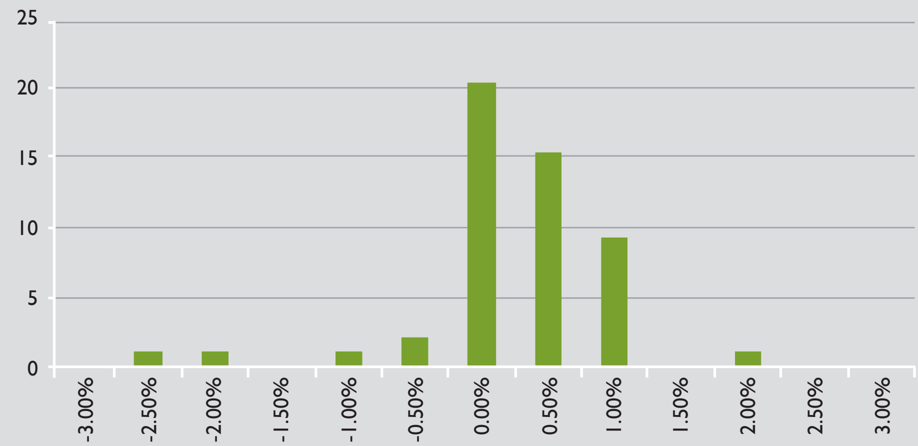
Strategy B: Precise tracking, larger impact

An alternative way to look at the problem is to consider **market impact as a budget** with an upper limit. The investor may feel that he may accept a market impact of 40 bp but not more. Looking at Exhibit 3 you immediately see that this budget corresponds to an annual tracking error of around 5bp. This means more precise tracking as above but it comes with a higher market impact.

Strategy C: Optimal utility

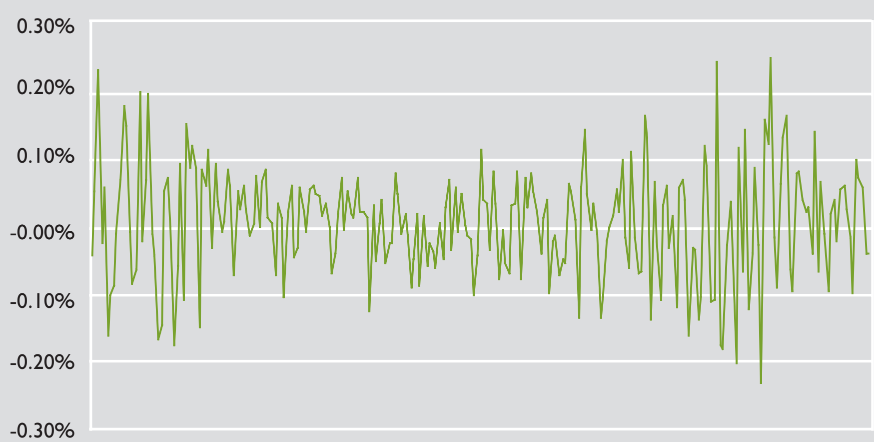
As was documented before, market impact and tracking error are two rather distinct measures. Market impact is a measure of potential loss that will occur with high probability when a given portfolio is traded, whereas tracking error is a symmetric risk measure. This means that both losses and gains are equally likely to happen. To obtain the optimal trade-off between these two measures, the investor needs to act according to his utility function, which, e.g., can be simply given as the (negative) sum of these two numbers. In this case, the optimal strategy would be to buy a portfolio with a

Histogram of active weights for strategy A (fair tracking, small impact) — Exhibit 4



Source: RiskLab GmbH

Daily active performance for strategy A (fair tracking, small impact) — Exhibit 5



Source: RiskLab GmbH

tracking error of 10.0bp and a market impact of 28.6, giving a total utility of -38.6. It can be seen from Exhibit 2 that all other choices would result in a lower utility.

We have illustrated the utility functions corresponding to strategies A, B and C in Exhibit 6. We see that the highest utility for strategy C is achieved with a portfolio with a tracking error of 10bp, the highest utility for strategy A is at the right hand side at 5bp, whereas the highest utility for strategy B is attained at the left hand side at 100bp.

SUMMARY

We have discussed the trade-off between two conflicting goals that a passive portfolio manager faces: Minimising tracking error while keeping transaction costs small. In section two, we have seen how market impact can be quantified and predicted. Section three has shown how this market impact model can be used in the context of portfolio optimisation. The basic idea here is that market impact is considered as the objective function which has to be minimised while tracking error figures as a constraint. Section four has shown how such an optimisation procedure can support the investment decision in passive portfolio management.

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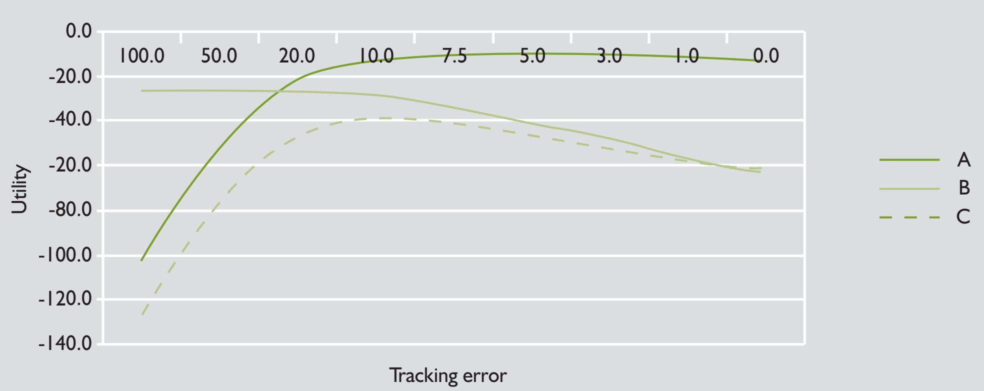
Utility functions for strategies A, B and C **Exhibit 6**



Source: RiskLab GmbH

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Notes:

1. For a detailed description of the model, see [Haf01]. For a related model, see [Bar00].
2. See [Min82] for a detailed description of the underlying algorithm

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