Fit for Leverage
-
Modelling of Asian Hedge Fund Returns

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Abstract

Hedge Funds typically reveal some statistical properties like serial correlation, non-normality, volatility clustering or leverage which have to be considered when risk positions of Hedge Funds are computed. We describe a simulation model based on a Markov-Switching autoregressive process that captures the specific features of Hedge Fund returns and allows especially for leverage effects in the data. The model is tested using publicly available Asian Hedge Fund Index data. We compare two different variants of the model by means of risk and performance measures. Our case study implies that if the leverage effect appears in the data, it is worth to fit for leverage in the parameter estimation process.

JEL classification: C22; G10; G15;
Keywords: Asian Hedge Fund; Leverage Effect; Markov-Switching Autoregressive Model;

1 Introduction

The Hedge Fund industry and especially the Asian Hedge Fund industry is clearly growing. The assets of Asia-focused Hedge Funds have grown fivefold over the years 2002 to 2005: from barely 20 billion USD in 2002 to an estimated 100 billion USD at the end of 2005. Some industry players believe the total could reach 250 billion USD by 2010 (see Gray (2006)). According to figures from 'Eurekahedge', a Singapore based research firm, there were nearly 660 Asia-focused Hedge Funds (HF) – including Japan and Australia – in November 2005, an increase of almost 20% from six months earlier.

HF often function as trigger and multiplier of crises and problems. For example, Asian HF played a notable role in the Asian Currency Crisis of 1997 (see, e.g., Fung and Hsieh (2000)). Another failure of appropriate risk management is the downturn of the US-based Long Term Capital Management (LTCM) HF in 1998 that was due to

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an underestimation of risk (see, e.g., Jorion (2000)). This illustrates the importance of a sound risk management and strongly emphasizes a risk-adjusted performance measurement.

In this paper we use two specifications of a Markov Switching Model to forecast the risk positions of HF. Markov Switching Models (MSM) have become increasingly popular in economic studies (see, e.g., Hamilton (1994), Billio and Pelizzon (2000), Timmermann (2000) or Brunner and Hafner (2006)) and are able to capture the typical statistical properties of HF, for example non-normality, serial correlation and volatility clustering. Another typical HF feature is the leverage effect. To the authors' best knowledge it is not yet captured by applications of Markov Switching Models in the literature. We extend the existing MSM of Brunner and Hafner (2006) by deriving the autocorrelation function of the leverage effect for the Method of Moments according to Timmermann (2000). Hence, we are able to analyse if it is worth to fit for the leverage effect that can be found in HF data.

The two main questions we address in this study are the following: can typical statistical properties of HF, as found in the US market (see, e.g., Lo (2001), Kat and Lu (2002) or Capocci and Hübner (2004)) also be confirmed for Asian HF? Is it worth including the leverage effect into a MSM used to predict the risk exposure or performance measures of HF?

The paper is organized as follows. In Section 2 we first describe the class of MSM in general and then derive two specifications for the parameter estimation. The various risk and performance measures are introduced in Section 3. Section 4 contains the statistical analysis of the empirical data from Asian HF Indices. In Section 5 the results of the two compared MSM specifications subject to the probability distributions and risk and performance measures are presented. Section 6 concludes.

2 Markov Switching Model

Since HF typically reveal some statistical properties like non-normality, autocorrelations, volatility clustering and leverage, a normal distribution is not appropriate to describe the evolution of HF returns. Instead, we use a MSM as introduced in Hamilton (1994). We extend the modelling approach of Brunner and Hafner (2006) so that it is not only able to capture non-normality, autocorrelations and volatility clustering, but also the leverage effect.

As time series processes can change dramatically over time, the idea of MSM is to model different states (or regimes) a time series process can be in. Each regime of a time series process is described by its own density function, which leads to the possibility of capturing typical features of HF returns.[1] The changes of regimes are modelled via a Markov chain $(S_t)_{t \in IR}$ with transition probabilities given by

$$P\{S_t = j \mid S_{t-1} = i, S_{t-2} = k, \ldots\} = P\{S_t = j \mid S_{t-1} = i\} = p_{ij} \tag{1}$$

where p_{ij} denotes the transition probability of changing from state i in period $t-1$ to state j in period t. The transition matrix is given by

[1] The joint density distribution reveals moments different from those of the single density functions, with especially the skewness and excess kurtosis being different from zero. Thus, the non-normality of HF returns is accounted for, while for example the feature of volatility clustering is captured by time-varying volatilities.

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1N} \\ p_{21} & \ddots & & \vdots \\ \vdots & & & \\ p_{N1} & \cdots & & p_{NN} \end{bmatrix}. \tag{2}$$

In the following, we take $N = 2$ (i.e. only two possible states) for a first-order Markov switching autoregressive time series process similar to Brunner and Hafner (2006). The return R_t at time t is given by

$$R_t = \mu_{S_t} + \Phi \cdot \left(R_{t-1} - \mu_{S_{t-1}}\right) + \sigma_{S_t} \cdot \varepsilon_t \quad , \tag{3}$$

where $|\Phi| < 1, \varepsilon_t \sim N(0,1), i.i.d.$. ε_t and S_t are assumed to be independent at all leads and lags and R_t is stationary. $S_t = 1$ if the process is in state 1, and $S_t = 2$ if it is in state 2.

Let $p_{11}, p_{22} < 1$ and $p_{11} + p_{22} > 0$ so that the Markov chain is ergodic, then there exists a unique stationary distribution $\boldsymbol{\pi}' = (\pi_1, \pi_2)$ given by

$$\pi_1 = \frac{p_{21}}{p_{12} + p_{21}} \text{ and } \pi_2 = \frac{p_{12}}{p_{12} + p_{21}} \quad . \tag{4}$$

The parameter vector

$$\boldsymbol{\theta} = (\mu_1, \mu_2, \sigma_1, \sigma_2, \Phi, p_{12}, p_{21}) \tag{5}$$

is estimated by moment matching. For this, we set up a system of seven equations to get an estimate for the seven parameters of $\boldsymbol{\theta}$.[2] We set up seven equations by equating the first to fourth centred moments (four equations) to the corresponding distribution moments as well as the autocorrelation of lag 1 to its empirical counterpart. The remaining equations can be chosen out of

- the autocorrelations of lag n , $n \in IN_0$,
- the autocorrelations of squared returns (volatility clustering) of lag m , $m \in IN_0$,
- and the autocorrelations between yesterday's returns and today's squared returns (leverage effect) of lag 1.

Again, the chosen autocorrelations are equated to the corresponding empirical autocorrelations.

The first to fourth centred moments and autocorrelations (serial correlation and volatility clustering) of a first-order Markov switching autoregressive process are given by (see Timmermann (2000) or Brunner and Hafner (2006)):

mean μ:

$$\mu = \boldsymbol{\pi}' \boldsymbol{\mu}_{\mathbf{S}} \tag{6}$$

variance σ^2:

[2] When the number of moment conditions is the same as the number of unknown parameters, the MoM estimator equals the Generalized Method of Moments (GMM) estimator, since the weighting matrix has no impact (see Hamilton (1994), Chapter 14).

$$\sigma^2 = \boldsymbol{\pi}'\left((\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) + \frac{\boldsymbol{\sigma}_S^2}{1-\Phi^2} \right) \tag{7}$$

skewness s:

$$\begin{aligned} s = \tfrac{1}{\sigma^3}\big[& \boldsymbol{\pi}'((\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1})) \\ & + 3\Phi^2 \boldsymbol{\pi}'\left(\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1})\right) \\ & + 3\boldsymbol{\pi}'\left((\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes \boldsymbol{\sigma}_S^2\right) \big] \end{aligned} \tag{8}$$

excess kurtosis ek:

$$\begin{aligned} ek = \tfrac{1}{\sigma^4}\big[& \boldsymbol{\pi}'((\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1})) \\ & + 6\boldsymbol{\pi}'\left((\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes \boldsymbol{\sigma}_S^2\right) \\ & + \boldsymbol{\pi}'(\mathbf{I}_2 - \Phi^4\mathbf{B})^{-1}\left(3\boldsymbol{\sigma}_S^4 + 6\Phi^2\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\sigma}_S^2\right) \\ & + 6\Phi^2\boldsymbol{\pi}'\left(\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1}) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1})\right) \big] - 3 \end{aligned} \tag{9}$$

autocorrelation of lag n $autocor_n$:

$$autocor_n = \frac{1}{\sigma^2}\left[\boldsymbol{\pi}'((\mathbf{B}^n(\boldsymbol{\mu}_S - \mu\mathbf{1})) \otimes (\boldsymbol{\mu}_S - \mu\mathbf{1})) + \Phi^n \boldsymbol{\pi}'(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right] \tag{10}$$

with $n \in IN_0$,

autocorrelation of squared returns lag m $autocorsq_m$:

$$autocorsq_m = \frac{E\left[\left(R_t^2 - E[R_t^2]\right)\left(R_{t-m}^2 - E[R_{t-m}^2]\right)\right]}{E\left[\left(R_t^2 - E[R_t^2]\right)^2\right]} \tag{11}$$

with

$$\begin{aligned} & E\left[\left(R_t^2 - E[R_t^2]\right)\left(R_{t-m}^2 - E[R_{t-m}^2]\right)\right] = \\ & = \Phi^{2m}\boldsymbol{\pi}'\left(\left((\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\mu}_S^2\right) \\ & + \Phi^{2m}\boldsymbol{\pi}'(\mathbf{I}_2 - \Phi^4\mathbf{B})^{-1}\left(3\boldsymbol{\sigma}_S^4 + 6\Phi^2\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\sigma}_S^2\right) \\ & + \boldsymbol{\pi}'\left(\left(\mathbf{B}^m(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\mu}_S^2\right) + \boldsymbol{\pi}'\left((\mathbf{B}^m\boldsymbol{\mu}_S^2) \otimes \boldsymbol{\mu}_S^2\right) \\ & + \boldsymbol{\pi}'\left(\sum_{i=1}^{m}\Phi^{2(m-i)}\left(\mathbf{B}^i(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2 + \mathbf{B}^i\boldsymbol{\mu}_S^2\right) \otimes \boldsymbol{\sigma}_S^2\right) \\ & + 4\Phi^m\boldsymbol{\pi}'\left(\left(\mathbf{B}^m(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}(\boldsymbol{\sigma}_S^2 \otimes \boldsymbol{\mu}_S)\right) \otimes \boldsymbol{\mu}_S\right) - \left(E[R_t^2]\right)^2 \end{aligned}$$

and

$$\begin{aligned} & E\left[\left(R_t^2 - E[R_t^2]\right)^2\right] = \\ & = \boldsymbol{\pi}'\boldsymbol{\mu}_S^4 + 6\Phi^2\boldsymbol{\pi}'\left(\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\mu}_S^2\right) \\ & + \boldsymbol{\pi}'(\mathbf{I}_2 - \Phi^4\mathbf{B})^{-1}\left(3\boldsymbol{\sigma}_S^4 + 6\Phi^2\left(\mathbf{B}(\mathbf{I}_2 - \Phi^2\mathbf{B})^{-1}\boldsymbol{\sigma}_S^2\right) \otimes \boldsymbol{\sigma}_S^2\right) \\ & + 6\boldsymbol{\pi}'\left(\boldsymbol{\mu}_S^2 \otimes \boldsymbol{\sigma}_S^2\right) - \left(E[R_t^2]\right)^2 \end{aligned}$$

with $m \in IN_0$.

In our approach we also include the autocorrelation between yesterday's returns and today's squared returns of lag *1* (leverage effect) $autocorle_1$:

$$autocorle_1 = \frac{E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1} - \mu\right)\right]}{\sqrt{E\left[\left(R_t^2 - E\left[R_t^2\right]\right)^2\right]} \cdot \sigma} \tag{12}$$

with

$$\begin{aligned} E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1} - \mu\right)\right] &= \\ &= \boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\boldsymbol{\mu}_\mathbf{S} - \mu\mathbf{1}\right)\right)\otimes \boldsymbol{\mu}_\mathbf{S}^2\right) + \boldsymbol{\pi}'\Phi^2\left(\mathbf{B}\left(\mathbf{I_2} - \Phi^2\mathbf{B}\right)^{-1}\left(\boldsymbol{\sigma}_\mathbf{S}^2 \otimes \left(\boldsymbol{\mu}_\mathbf{S} - \mu\mathbf{1}\right)\right)\right) \\ &\quad + \boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\boldsymbol{\mu}_\mathbf{S} - \mu\mathbf{1}\right)\right)\otimes \boldsymbol{\sigma}_\mathbf{S}^2\right) + 2\boldsymbol{\pi}'\Phi\left(\left(\mathbf{B}\left(\mathbf{I_2} - \Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^2\right)\otimes \boldsymbol{\mu}_\mathbf{S}\right) \end{aligned}$$

where $\boldsymbol{\pi} = \begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix}$, $\boldsymbol{\mu}_\mathbf{S} = \begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}$, $\mathbf{1} = \begin{pmatrix}1\\ 1\end{pmatrix}$, $\boldsymbol{\sigma}_\mathbf{S} = \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}p_{11} & p_{12}\\ p_{21} & p_{22}\end{pmatrix} = \begin{pmatrix}1-p_{12} & p_{12}\\ p_{21} & 1-p_{21}\end{pmatrix}$,[3] $\mathbf{I_2} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$, Φ is the autocorrelation parameter, R_t the HF return in period *t* and $\otimes$ the element by element multiplication.

Hence, the parameter vector $\boldsymbol{\theta}$ (see (5)) is estimated via

$$\begin{aligned} \min_\theta \Big\{ &(\tilde{\mu} - \mu)^2 + (\tilde{\sigma} - \sigma)^2 + (\tilde{s} - s)^2 + (\tilde{e}k - ek)^2 \\ &+ \sum_{i=1}^{n}\left(\tilde{a}utocor_i - autocor_i\right)^2 + \sum_{j=1}^{m}\left(\tilde{a}utocorsq_j - autocorsq_j\right)^2 \\ &+ \left(\tilde{a}utocorle_p - autocorle_p\right)^2 \Big\} \end{aligned} \tag{13}$$

with $n + m + p = 3$ and where $\tilde{\mu}$ resembles the sample mean, $\tilde{\sigma}^2$ the sample variance, $\tilde{s}$ the sample skewness, $\tilde{e}k$ the sample excess kurtosis, $\tilde{a}utocor_n$ $(n \in IN_0)$ the sample autocorrelations of lag n, $\tilde{a}utocorsq_m$ $(m \in IN_0)$ the sample autocorrelations of squared returns of lag m and $\tilde{a}utocorle_p$ $(p \in \{0,1\})$ the sample autocorrelations of yesterday's returns and today's squared returns if $p = 1$, while $p = 0$ indicates that this number is not considered.

We tested different specifications of (13) and found the best results when using autocorrelation of lag 1, volatility clustering and autocorrelation of lag 2 (see also Brunner and Hafner (2006)) or autocorrelation of lag 1, volatility clustering and leverage effect. Hence, we will focus on these two variants in the empirical part.

3 Risk and performance measures

In this section, an overview on some well-known risk and performance measures is given. We differentiate between tail related risk measures like Value at Risk and Conditional Value at Risk and performance measures based on the whole distribution like Sharpe Ratio, Adjusted Sharpe Ratio, Omega and Sharpe-Omega.

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[3] The matrix $\mathbf{B}$ gives the transition probabilities for the "time-reversed" Markov chain that moves back in time. In the case with two states, the "backward" transition probability matrix $\mathbf{B}$ equals the "forward" transition probability matrix $\mathbf{P}$. For more details see Timmermann (2000), p. 87.

Value at Risk

The Value at Risk VaR_α of a random variable R to a confidence level $1-\alpha$ is defined by (see, e.g., p. 252 of Zagst (2002))

$$VaR_\alpha(R) = E(R) - R_\alpha \,. \tag{14}$$

where

$$R_\alpha = \sup\{x \in IR : P(R < x) \le \alpha\} \tag{15}$$

is the value of R that will be exceeded with probability $1-\alpha$.

Despite its popularity, VaR has some negative properties: it does not address the distribution of potential losses on those rare events when the VaR estimate VaR_α is exceeded. Furthermore, it is no coherent risk measure (see, e.g., p. 254 of Zagst (2002)). A risk measure that addresses both disadvantages is the Conditional Value at Risk.

Conditional Value at Risk

The Conditional Value at Risk (CVaR) is a risk measure that focuses on the losses which exceed VaR. In the literature, CVaR is also referred to as Expected Shortfall.[4]

$CVaR_\alpha$ to a predefined confidence level $1-\alpha$ is defined as the average loss given that VaR_α is exceeded (see, e.g., p. 263 of Zagst (2002)):

$$CVaR_\alpha(R) = -E(R \mid R \le R_\alpha), \tag{16}$$

where R_α is given by (15).

It can be easily seen that CVaR is at least as great as VaR, and usually it will be greater. As already mentioned, CVaR is a coherent risk measure (see, e.g., Acerbi and Tasche (2002)) and therefore more appropriate than VaR when assessing the risk of a portfolio.

Sharpe Ratio

A popular performance measure is the Sharpe Ratio (SR) introduced by William F. Sharpe in 1966. It measures the risk-adjusted performance of an investment or a trading strategy relative to a benchmark asset, such as the risk-free rate of return.

The SR of an asset return R, $SR(R)$, is defined as the expected excess return per unit of risk associated with the excess return:

$$SR(R) = \frac{E(R) - r_f}{STD(R)} = \frac{\mu_R - r_f}{\sigma_R} \tag{17}$$

where the expected excess return is given as the expected asset return $E(R) = \mu_R$ beyond the risk free rate of return r_f and the risk is given by the standard deviation of R, $STD(R) = \sigma_R$.

The SR only is an appropriate performance measure when the return distribution solely depends on two parameters: location and scale parameter. Thus the SR does

[4] Note that this is only true for continuous random variables. For more details and proof see Acerbi and Tasche (2002), p. 10.

not measure correctly the performance of non-normally tailed or skewed return distributions, such as those of fat tailed and negatively skewed HF returns. Therefore, a more generalized SR, called the Adjusted Sharpe Ratio, is presented.

Adjusted Sharpe Ratio

The Adjusted Sharpe Ratio (ASR) extends the SR by taking non-normality in form of skewness and excess kurtosis of the asset returns into account.

The ASR of an asset return R, $ASR(R)$, is given by (see, e.g., p. 44 of Alexander and Sheedy (2004))

$$ASR(R) = SR(R) \cdot \left[1 + \frac{s(R)}{6} SR(R) - \frac{ek(R)}{24} SR(R)^2\right] \quad (18)$$

where $s(R)$ and $ek(R)$ represent the skewness and excess kurtosis of an asset return R, respectively.

Although the ASR is more appropriate than the SR for measuring the performance of non-normally distributed asset returns, the ASR still does not capture all the information – only the first four moments – contained in the return series.

Omega

The Omega measure introduced by Keating and Shadwick (2002) was developed with the intention to take the entire return distribution into account and is defined as the ratio of the gain with respect to a threshold *L* and the loss with respect to the same threshold. Then Omega Ω_L with respect to a threshold *L* is given by [5]

$$\Omega_L = \frac{\int_L^b [1 - F(r)] dr}{\int_a^L F(r) dr} \quad (19)$$

where (a,b) is the interval of returns and F is the cumulative distribution of returns.

Sharpe-Omega

A new version of Omega introduced by Kazemi *et al.* (2003) is the so called Sharpe-Omega given by

$$S\Omega_L = \frac{E(R) - L}{\int_a^L F(r) dr} = \Omega_L - 1. \quad (20)$$

The formula for Sharpe-Omega consists of the numerator of (17), where the risk-free rate of return r_f is replaced by the threshold *L*, and the denominator of (19).

Since the numerator of (20) corresponds to the price of a put option, Sharpe-Omega represents a measure of risk/return that is more intuitive than Omega. Since the price of the put option is the cost of protecting an investment's return below the target ratio (given by *L*), it is a reasonable measure of the investment's riskiness.

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[5] In fact, the evaluation of an investment with the Omega function should be considered for thresholds between 0% and the risk free rate, see Bacmann and Scholz (2003), p. 3.

4 Statistical properties of Asian Hedge Funds

This section aims to characterize some of the typical features found in HF return time series. Therefore, we first give a short summary on the statistical properties. Then, we analyse if these properties also apply for our Asian HF data.

Compared to stocks and bonds, HF reveal some typical statistical properties, which have been confirmed by a number of studies, for example Kat and Lu (2002). Main statistical properties of HF, as for example pointed out in Brunner and Hafner (2006), are:

- *Non-normality:* HF time series are characterized by negatively skewed and fat tailed returns. Skewness is defined as the degree of asymmetry of a probability distribution. A negative skewness implies that the left tail is the longest and that the mass of the distribution is concentrated on the right side of the density function. Kurtosis is defined as the fatness of the tails of a probability distribution. A normally distributed random variable has an excess kurtosis of zero. Now, a positive excess kurtosis implies fatter tails, meaning that extreme or tail events are more likely.
- *Autocorrelation:* In contrast to long-only equity portfolios and mutual funds, HF returns exhibit in most cases strong serial correlation (see, e.g., Getmansky *et al.* (2004). Positive serial correlation or autocorrelation means that today's HF returns depend on last periods' HF returns. When today's returns only depend on yesterday's returns, we speak of first-order autocorrelation; when today's returns depend on the returns two (three) periods ago, we speak of second-order (third-order) autocorrelation.
- *Volatility Clustering:* Additionally, HF time series often exhibit volatility clustering, which means that large changes in prices tend to follow large changes in prices, of either sign, and small changes tend to follow small changes. While the returns themselves may be uncorrelated, absolute returns or their squares can be positively autocorrelated. This means that volatility is dependent upon past realizations of the volatility process.

As already mentioned above, our special focus in this paper is on an additional property, the leverage effect (see, e.g., Alexander (2005) for a theoretical description and Bae *et al.* (2007) for an empirical study).

- *Leverage Effect:* The leverage effect implies that volatility is higher in a falling market than in a rising market. The reason for this may be that when the equity price falls, the debt remains constant in the short term. So the debt-equity ratio increases, the firm becomes more leveraged, the future of the firm becomes more uncertain and the equity price therefore becomes more volatile.

For the empirical part of this paper we use publicly available HF Index data from 'Eurekahedge', a hedge fund research company based in Singapore.[6] We focus on those eight HF Index return time series (monthly data, ranging form December 1999 to June 2006) that include the whole region Asia/Pacific (see Figure 1).

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[6] For more information on the HF Indices see www.eurekahedge.com.

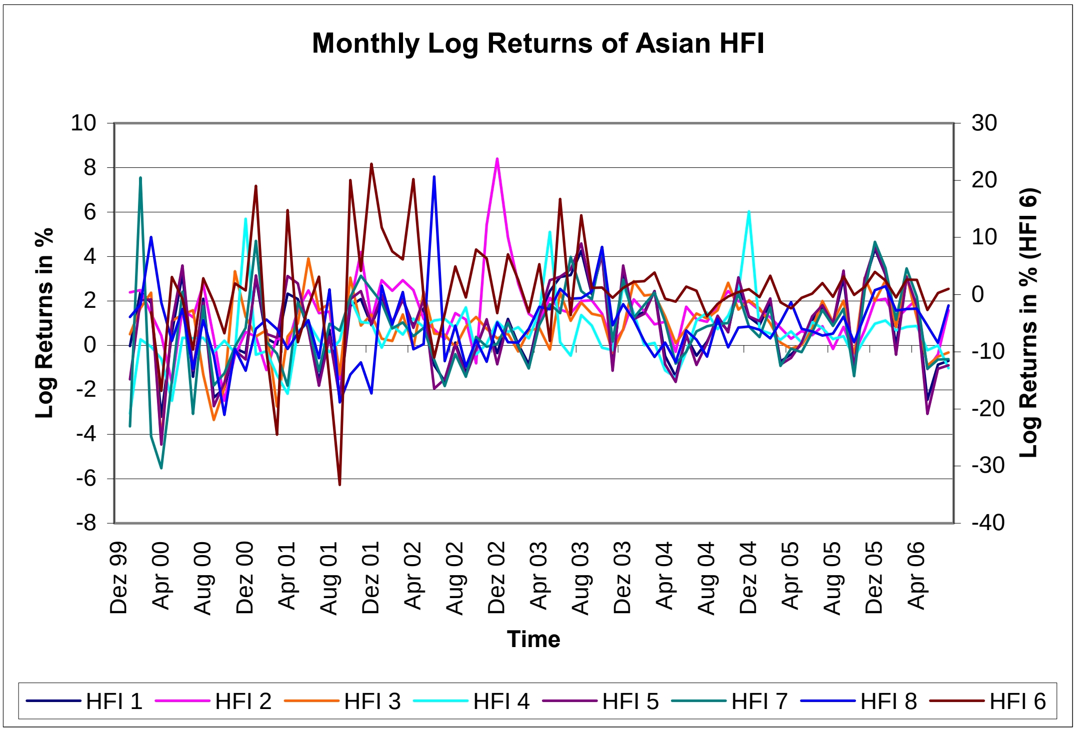

Figure 1: Monthly Log Returns of Asian HFI

	Basic Statistics						Jarque-Bera Test	
	mean (in %)	std.dev. (in %)	skewness	excess kurtosis	max (in %)	min (in %)	Statistic	p-value
HFI 1	0.872	1.707	-0.131	-0.639	4.343	-3.202	1.317	51.76%
HFI 2	1.326	1.552	1.164	4.839	8.403	-2.493	101.002	0.00%
HFI 3	0.923	1.434	-0.955	1.995	3.916	-4.285	27.003	0.00%
HFI 4	0.570	1.400	1.346	5.070	6.027	-3.067	115.101	0.00%
HFI 5	0.849	1.921	-0.246	-0.527	4.587	-4.464	1.512	46.95%
HFI 6	0.783	8.745	-0.551	3.216	22.911	-33.28	40.246	0.00%
HFI 7	0.751	2.046	-0.003	1.460	7.548	-5.519	8.073	1.77%
HFI 8	0.820	1.554	0.980	3.987	7.601	-3.118	69.499	0.00%

Table 1: Basic Statistics and Jarque-Bera Test for Asian HFI

Table 1 gives a summary of the basic statistics for the Asian HF Indices (HFI). Five out of eight HFI exhibit a negative skewness, and the excess kurtosis is mainly positive. From the Jarque-Bera Test (see the last two columns of Table 1) we see that the null hypothesis of normally distributed returns can be rejected for six Asian HFI at the 5% significance level.

As a result, we can say that most Asian HFI are non-normally distributed. Thus, the typical HF feature of non-normality also applies for Asian HFI returns.

The empirical autocorrelations, volatility clustering and leverage effect are shown in Table 2. Most HF are positively serially correlated, and the absolute values are gradually decreasing. Volatility clustering ranges from -0.018 to 0.521, whereas the leverage effect ranges from -0.321 to 0.426. All three statistical properties of Table 2 can be statistically confirmed with the Ljung-Box Test for autocorrelations, and thus are in line with previous studies (see, e.g., Getmansky *et al.* (2004)).

To sum it up, our empirical HF data on Asian HFI reveal the typical HF features: non-normality, serial correlation, volatility clustering and leverage effect.

	autocorrelation of lag		**volatility clustering**	**leverage[7] effect**
	1	**2**		
HFI 1	0.152	0.126	0.159	0.255
HFI 2	0.464	0.075	0.521	0.426
HFI 3	0.106	-0.011	0.014	-0.056
HFI 4	0.179	-0.011	-0.005	0.068
HFI 5	0.135	0.086	0.100	0.255
HFI 6	-0.041	0.001	0.284	-0.321
HFI 7	0.084	0.087	0.313	-0.100
HFI 8	0.146	0.246	-0.018	0.033

Table 2: Autocorrelations, volatility clustering and leverage effect of Asian HFI

5 Empirical Comparison of Markov Switching Models

In this section we want to compare the two different variants of parameter estimation introduced in Section 2 and their impact on risk and performance measures. The first one ('MSM lag 2') is similar to the one in Brunner and Hafner (2006), while the second one ('MSM le') takes the leverage effect instead of the autocorrelation of lag 2 into account. Both of them take account for non-normality by including skewness and excess kurtosis and furthermore, the two HF features volatility clustering and autocorrelation of lag 1 (see Table 3).

[7] For stock returns the leverage effect has to be negative as low stock returns are associated with higher volatilities (see, e.g., Alexander (2005) or Bae *et al.* (2007)). In contrast to this, Hedge Funds can follow various investment strategies, e.g. short equities or long commodities, which might result in a positive value for the leverage effect coefficient. Nevertheless, the economic interpretation is the same as for the case of stock returns.

	MSM lag2	MSM le
non-normality	+	+
volatility clustering	+	+
autocorrelation lag 1	+	+
autocorrelation lag 2	+	
leverage effect		+

Table 3: Survey of MSM models and incorporated features

For both sets of estimated parameters, we simulate the evolution of the HFI return time series for 60 months, using a Monte Carlo Simulation with 5.000 paths.

Comparison of risk and performance measures

Then we look at the probability distribution of the return series after five years. More detailed, we look at the risk and performance measures (VaR, CVaR, SR, ASR, Omega and Sharpe-Omega, see Section 3) of the return distributions and compare these figures to the empirical risk and performance measures.

Table 4 shows the empirical risk and performance measures as well as those of the simulated ‘MSM lag2’ and ‘MSM le’, respectively. The last row additionally contains the ratio of empirical leverage to empirical autocorrelation of lag 2 (‘Ratio le/lag2’) as an indicator for the severity of leverage in the data.

We concentrate on the 5% VaR and CVaR, respectively, because conclusions drawn from the 1% measures could be misleading as our empirical return series are rather small. In half the cases for the risk measures VaR and CVaR, the ‘MSM le’ outperforms ‘MSM lag2’. Thereby, we can see that the more distinct the leverage effect compared to the autocorrelation of lag 2, the more often tend VaR and CVaR measures of ‘MSM le’ to be closer to the empirical values than the corresponding ‘MSM lag2’ measures.

For the remaining performance measures, which take the whole distribution into account, the specification ‘MSM le’ is clearly preferable, since it yields – except for HFI 7 – the smaller differences to the empirical performance measures. Thus, we can conclude that ‘MSM le’ is better in forecasting the performance measures SR, ASR, Omega and Sharpe-Omega.

Comparison of simulated and empirical distribution functions

We also apply the chi-squared test of goodness of fit (see, e.g., Bamberg *et al.* (2007)) to compare the different simulated HF return distributions, and, on the other hand to compare the left tail of the simulated HF return distributions for the ‘MSM le’ and ‘MSM lag2’ to each other.

The chi-squared test of goodness of fit tests if an unknown distribution resembles a given hypothetical distribution. The smaller the test statistic, the better the unknown distribution (i.e. the simulated distribution) resembles the hypothetical one (i.e. the empirical distribution).

		HFI 1	HFI 2	HFI 3	HFI 4	HFI 5	HFI 6	HFI 7	HFI 8
VaR 5%	Empirical	-1.98	-1.07	-1.40	-1.36	-2.01	-12.0	-2.01	-1.16
	MSM lag2	-2.14	-1.15	-2.00	-1.99	-2.23	-12.4	-2.42	-1.52
	MSM le	-1.74	-1.00	-1.79	-1.81	-2.17	-12.9	-2.54	-1.89
CVaR 5%	Empirical	-2.59	-1.67	-3.07	-2.30	-3.17	-22.4	-4.08	-2.29
	MSM lag2	-2.73	-2.25	-2.65	-3.20	-3.03	-17.8	-3.58	-2.83
	MSM le	-2.33	-2.02	-2.45	-3.03	-2.81	-18.6	-3.76	-3.20
SR	Empirical	0.36	0.69	0.47	0.23	0.31	0.06	0.24	0.37
	MSM lag2	0.34	0.65	0.41	0.19	0.27	0.08	0.26	0.35
	MSM le	0.35	0.67	0.42	0.22	0.32	0.07	0.22	0.35
ASR	Empirical	0.36	0.72	0.43	0.24	0.31	0.06	0.24	0.38
	MSM lag2	0.33	0.63	0.39	0.19	0.28	0.08	0.26	0.34
	MSM le	0.35	0.66	0.42	0.22	0.32	0.07	0.22	0.35
Omega	Empirical	2.42	7.22	3.36	2.10	2.11	1.21	1.93	2.89
	MSM lag2	2.21	5.69	2.61	1.87	1.99	1.24	1.96	2.60
	MSM le	2.43	6.20	2.80	1.99	2.22	1.21	1.78	2.93
Sharpe-Omega	Empirical	1.42	6.22	2.36	1.10	1.11	0.21	0.93	1.89
	MSM lag2	1.21	4.69	1.61	0.87	0.99	0.24	0.96	1.60
	MSM le	1.43	5.20	1.80	0.99	1.22	0.21	0.78	1.93
Ratio le/lag2		*2.03*	*5.68*	*5.14*	*6.12*	*2.97*	*427.8*	*1.15*	*0.13*

Table 4: Comparison of risk and performance measures

The results for the tests performed on the entire simulated distribution (‘entire’) and on the left tail of the simulated distribution (‘tails’) are presented in Table 5. In each case, the specification (lag2 for ‘MSM lag2’ and le for ‘MSM le’) is named, which has the lowest test statistic. Hence, Table 5 gives the specification that rebuilds most appropriate the entire empirical return distribution or the tails of the empirical return distribution, respectively. The HFI are arranged according to the ratio of leverage effect to autocorrelation lag 2.

	HFI 6	HFI 4	HFI 2	HFI 3	HFI 5	HFI 1	HFI 7	HFI 8
entire	le	lag2	lag2	le	lag2	lag2	lag2	lag2
tails	le	le	le	le	lag2	le	lag2	le
Ratio le/lag2	*427.88*	*6.12*	*5.68*	*5.14*	*2.97*	*2.03*	*1.15*	*0.13*

Table 5: Preferred specifications (‘MSM lag2’ or ‘MSM le’) after the distribution fit

If we look at the entire distributions, ‘MSM le’ is only two times preferred, but both cases correspond to HFI that have a high ‘Ratio le/lag2’ and the difference between the test statistics for HFI 2 is quite small. This may be an indication for the fact that the more distinct the leverage effect compared to the autocorrelation lag 2, ‘MSM le’ tends to simulate the entire return distribution better than ‘MSM lag2’.

In contrast, ‘MSM le’ is superior to ‘MSM lag2’ regarding the question, which MSM specification fits the tails of the empirical distribution function better. This result supports the above observation that VaR and CVaR measures of ‘MSM le’ tend to be closer to the empirical values than the corresponding ‘MSM lag2’ measures the larger the ratio le/lag2.

6 Conclusion

As already pointed out in the introductory section, a sound risk management for HF is essential to anticipate the risk exposure. Therefore we extended the MSM model of Brunner and Hafner (2006) by including the leverage effect that is a typical HF feature with impact on the risk exposure. Then we compared the two MSM specifications (one with leverage effect ‘MSM le’, the other without ‘MSM lag2’) to see which specification is better to forecast risk and performance measures.

The following results can be stated:

- The typical HF features non-normality, serial correlation, volatility clustering and leverage effect apply for Asian HFI.
- The larger the relative differences between leverage effect and autocorrelation lag 2, the better the specification ‘MSM le’ tends to be. This is indicated by the comparison of the various risk and performance measures, and on the other hand confirmed by the chi-squared test of goodness of fit.
- ‘MSM le’ clearly reproduces better estimates for the performance measures SR, ASR, Omega and Sharpe-Omega than ‘MSM lag2’.
- Additionally, ‘MSM le’ is also better to fit the tails of the probability distribution function to the empirical one.

It is therefore worth to fit the model for the leverage effect, especially if the ratio ‘le/lag2’ is large.

For future prospects, one could try to verify these results with longer empirical time series on Asian HFI, Asian single HF and perhaps also for Asian Fund of Funds.

Appendix

A. Proof of Markov Regime Switching formula

For sake of completeness we derive the formula of the leverage effect (Equation (13)). This is similar to the derivation of the autocorrelation formulas (see, e.g., Timmermann (2000)).

The underlying process is given by Equation (3). Further assumptions are:

1. The process starts from its steady-state distribution.
2. The error terms ε_t are iid.
3. ε_t and S_t are independent at all leads and lags.
4. R_t is stationary.

Using (3) we get for the squared return R_t^2 at period t

$$\begin{aligned} R_t^2 &= \left(\mu_{S_t} + \Phi\left(R_{t-1} - \mu_{S_{t-1}}\right) + \sigma_{S_t}\varepsilon_t\right)^2 \\ &= \mu_{S_t}^2 + \Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^2 + \sigma_{S_t}^2\varepsilon_t^2 + 2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right) \\ &+ 2\mu_{S_t}\sigma_{S_t}\varepsilon_t + 2\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right)\sigma_{S_t}\varepsilon_t \end{aligned} \tag{21}$$

Noting, that linear terms of ε_t will be uncorrelated with terms dated period t or earlier, the last line of (21) can be omitted when inserting (21) into the following equation:

$$\begin{aligned} &E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1} - \mu\right)\right] = \\ &= E\left[\left(\mu_{S_t}^2 + \Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^2 + \sigma_{S_t}^2\varepsilon_t^2 + 2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right) - E\left[R_t^2\right]\right)\cdot\right. \\ &\left.\cdot\left(\left(R_{t-1} - \mu_{S_{t-1}}\right) + \left(\mu_{S_{t-1}} - \mu\right)\right)\right] \\ &= E\left[\mu_{S_t}^2\left(R_{t-1} - \mu_{S_{t-1}}\right)\right] + E\left[\mu_{S_t}^2\left(\mu_{S_{t-1}} - \mu\right)\right] + E\left[\Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^3\right] \\ &+ E\left[\Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^2\left(\mu_{S_{t-1}} - \mu\right)\right] + E\left[\sigma_{S_t}^2\varepsilon_t^2\left(R_{t-1} - \mu_{S_{t-1}}\right)\right] \\ &+ E\left[\sigma_{S_t}^2\varepsilon_t^2\left(\mu_{S_{t-1}} - \mu\right)\right] + E\left[2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right)^2\right] \\ &+ E\left[2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right)\left(\mu_{S_{t-1}} - \mu\right)\right] \\ &- E\left[E\left[R_t^2\right]\left(R_{t-1} - \mu_{S_{t-1}}\right)\right] - E\left[E\left[R_t^2\right]\left(\mu_{S_{t-1}} - \mu\right)\right] \end{aligned} \tag{22}$$

where all covariance terms that include ε_t, S_t or $\left(R_t - \mu_{S_t}\right)$ with odd exponents cancel out. This follows from assumptions 2 and 3. The last two terms of (22) can also be deleted, due to the fact that $\mu = E\left[R_t\right] = \boldsymbol{\pi}'\boldsymbol{\mu}_S$ (for proof see Timmermann (2000)).

Hence, (22) reduces to

$$\begin{aligned} &E\left[\left(R_t^2 - E\left[R_t^2\right]\right)\left(R_{t-1} - \mu\right)\right] = \\ &= \underbrace{E\left[\mu_{S_t}^2\left(\mu_{S_{t-1}} - \mu\right)\right]}_{T1} + \underbrace{E\left[\Phi^2\left(R_{t-1} - \mu_{S_{t-1}}\right)^2\left(\mu_{S_{t-1}} - \mu\right)\right]}_{T2} \\ &+ \underbrace{E\left[\sigma_{S_t}^2\varepsilon_t^2\left(\mu_{S_{t-1}} - \mu\right)\right]}_{T3} + \underbrace{E\left[2\mu_{S_t}\Phi\left(R_{t-1} - \mu_{S_{t-1}}\right)^2\right]}_{T4}, \end{aligned} \tag{23}$$

where terms T1 and T3 are given by

$$T1 = \boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\boldsymbol{\mu}_\mathbf{S} - \mu\mathbf{1}\right)\right) \otimes \boldsymbol{\mu}_\mathbf{S}^\mathbf{2}\right) \tag{24}$$

$$T3 = \boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\boldsymbol{\mu}_\mathbf{S} - \mu\mathbf{1}\right)\right) \otimes \boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right) \tag{25}$$

To derive expressions for the second and forth terms T2 and T4, notice that

$$
\begin{aligned}
E\left[\left(\mathbf{R}_{t-1}-\boldsymbol{\mu}_{S_{t-1}}\right)^2 \mid S_t\right] &= \mathbf{B}\cdot E\left[\left(\mathbf{R}_{t-1}-\boldsymbol{\mu}_{S_{t-1}}\right)^2 \mid \mathbf{S}_{t-1}\right] \\
&= \mathbf{B}\cdot\sum_{i=0}^{\infty}\Phi^{2i}\mathbf{B}^i\boldsymbol{\sigma}_S^2 \\
&= \mathbf{B}\left(\mathbf{I}_2-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_S^2
\end{aligned} \tag{26}
$$

where $\mathbf{I}_2$ is the 2-dimensional identity matrix and $\mathbf{B}$ the backward transition probability matrix.[12] The third equation follows from the property of a geometric series and the fact that $\left(\mathbf{I}_2-\Phi^2\mathbf{B}\right)$ is invertible. Since $|\Phi|<1$ this will automatically be satisfied for all transition probability matrices since $\mathbf{B}$ has a single eigenvalue equal to unity and its remaining eigenvalues are less than one.

Using (26), we get

$$
T2=\boldsymbol{\pi}'\Phi^2\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\left(\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\right)\right) \tag{27}
$$

$$
T4=2\boldsymbol{\pi}'\Phi\left(\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right)\otimes\boldsymbol{\mu}_\mathbf{S}\right) \tag{28}
$$

Inserting (24), (25), (27) and (28) into (23), we receive the nominator of the leverage formula (12).

To derive the denominator of (12), we use the variance formula

$$
\sigma^2=E\left[\left(R_t-\mu\right)^2\right]=\boldsymbol{\pi}'\left(\left(\boldsymbol{\mu}_S-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_S-\mu\mathbf{1}\right)+\frac{\boldsymbol{\sigma}_S^2}{1-\Phi^2}\right), \tag{29}
$$

and the formula for the fourth centred moment (for a proof see Timmermann (2000))

$$
\begin{aligned}
E\left[\left(R_t-\mu\right)^4\right] &= \boldsymbol{\pi}'\left(\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\right) \\
&\quad+6\boldsymbol{\pi}'\left(\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right) \\
&\quad+\boldsymbol{\pi}'\left(\mathbf{I_2}-\Phi^4\mathbf{B}\right)^{-1}\left(3\boldsymbol{\sigma}_\mathbf{S}^4+6\Phi^2\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right)\otimes\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right) \\
&\quad+6\Phi^2\boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\otimes\left(\boldsymbol{\mu}_\mathbf{S}-\mu\mathbf{1}\right)\right).
\end{aligned}
$$

Setting $\mu=0$ we obtain

$$
\begin{aligned}
E\left[\left(R_t^2-E\left[R_t^2\right]\right)^2\right] &= E\left[R_t^4\right]-\left(E\left[R_t^2\right]\right)^2 \\
&= \boldsymbol{\pi}'\boldsymbol{\mu}_\mathbf{S}^4+6\boldsymbol{\pi}'\left(\boldsymbol{\mu}_\mathbf{S}^\mathbf{2}\otimes\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right) \\
&\quad+\boldsymbol{\pi}'\left(\mathbf{I_2}-\Phi^4\mathbf{B}\right)^{-1}\left(3\boldsymbol{\sigma}_\mathbf{S}^4+6\Phi^2\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right)\otimes\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right) \\
&\quad+6\Phi^2\boldsymbol{\pi}'\left(\left(\mathbf{B}\left(\mathbf{I_2}-\Phi^2\mathbf{B}\right)^{-1}\boldsymbol{\sigma}_\mathbf{S}^\mathbf{2}\right)\otimes\boldsymbol{\mu}_\mathbf{S}^\mathbf{2}\right)-\left(E\left[R_t^2\right]\right)^2
\end{aligned} \tag{30}
$$

which completes the proof.

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[12] For more details see Footnote 3 or Timmermann (2000), p. 87.

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