Financial Study

Assessing Investment Strategies

for Defined Contribution Pension Plans

under various Payout Options

(A Background Paper to the OECD Policy Report)

Authors:

Dr. Gerhard Scheuenstuhl, risklab GmbH

Dr. Sandra Blome, Institut für Finanz- und Aktuarwissenschaften

Dr. Wolfgang Mader, risklab GmbH

David Karim, risklab GmbH

Tim Friederich, risklab GmbH

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risklab GmbH Seidlstraße 24-24a 80335 München

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1 Introduction

In the wake of the recent financial crisis, a number of pension funds and governmental authorities are trying to find a scientifically sound answer to the question whether a long term shift from traditional Defined Benefit (DB) pension systems to a Defined Contribution (DC) pension system would introduce too much risk into the pension system. DC plans constitute already a large part of total pension income in some countries like Chile or Mexico in South America or Poland and Lithuania in Eastern Europe. The Organization for Economic Cooperation and Development (OECD) therefore has called for new rules, claiming that regulations should be designed such as to limit some of these risks and avoid situations where older workers and retirees are exposed to major losses in their retirement income.

We have to acknowledge the fact that in today's corporate practice we find a vast variety of different DC plan arrangements. This affects the type of investment concept used for the savings as well as for example the design of payment phases (also called payout options). Hence, plan members face a challenging decision. There is a case for protecting DC pension income from large market shocks, at least in default options designed for people who do not make an active choice of investment.

This study aims at providing a systematic and quantitative basis for assessing the risk-return characteristics of DC pension plans and the retirement income generated by these. Since there is not a single correct measure for expressing the risk-retirement income trade-off that could be used as a guide for public policy decisions, our analysis will not only look into different risk-return measures but it will also assess and compare different investment solutions regarding their downside protection and their upside potential. As risk is depending on the individual view point, risk measures can be defined in many different ways and may be rather complex. The risk measures used in the study are applied to assess retirement income results but they are not designed for the communication with plan members and retirees.[1]

In order to achieve this goal, we will investigate the impact of different long-term investment strategies used in actual pension plans (with a special focus on life-cycle strategies) and regarding their equity glide paths. We will also assess their suitability during both the accumulation and payout stage of a DC system. This assessment is based on a characterization of the resulting risk-return profiles for the retirees. We will then focus on selected investment strategies, which have similar risk-return

[1] Communication of the risk involved to plan members is of course a very important task as it holds also for any recommendations on the consequences regarding contribution payments. However, this whole issue covers a complete study itsself and, hence, cannot be in the scope of these analyses.

characteristics but present different schemes or avoiding risk. The study sets up three categories of investment strategies in order to represent high, medium and low risk profiles.

The analysis starts with a description of the underlying DC pension plan. After introducing the framework used to portray the savings phase, Section 2 of the study will highlight the different payout options we examined. Together with different investment strategies (introduced in Section 3), these options make up the financial simulation model. Investment returns will be measured during both the savings and the payout phase. The replacement rate distribution will be used as the main outcome figure to determine these measures. Section 4 will also introduce the different risk-return measures used to determine the suitability of the researched investment strategies. Section 5 will underline the main outcomes of this study for all the payout options and provide the first guidelines drawn from the results. Statistics on replacement rate distributions will be represented as percentile plots in order to assess both the downside and the upside potential of investment returns. The simulation results will be used to compare investment strategies that have a similar risk avoidance profile. Finally Section 6 will summarize the discussion, which should serve as a structured basis for further debate and conclusions as far as policy recommendations are concerned.

2 Projection Model for DC Pension Plans

In this Section we define a set of DC plans, which will be the common basis for the further analyses of this study. Considering the large number of variations in DC plan design, this study addresses only a few typical plan designs, which are represented in many international DC pension markets.

The analysis is set up to illustrate a practical situation. Stochastic projections of the potential retirement income will be used to assess the results of DC plans. Realistic projections require a detailed specification of the plans' characteristics. We use the following main components for the projection model:

- Basic plan structure and characteristics of the savings phase.
- Payout option during decumulation or retirement phase.
- Investment strategies describing how savings are invested in financial assets.

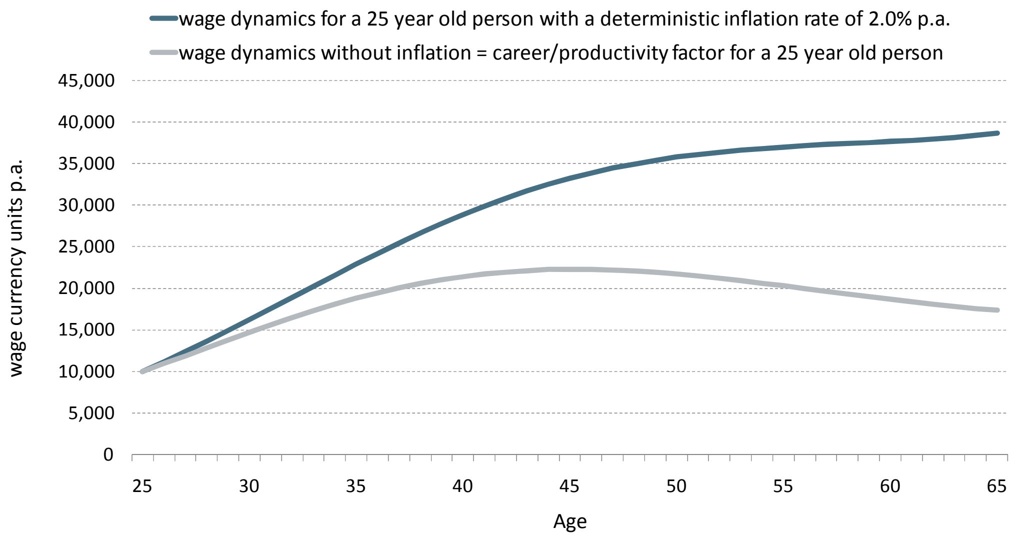
Given our focus on the design of investment strategies and our interest in assessing their quality as default options, we will analyze 23 different investment strategies, reflecting typical strategies used in DC plans. Since the assessment of investment strategies is affected by the form of the underlying payout option, we also consider a variety of 5 typical payout designs. The underlying structure of the savings period has only minor influence on the quality of the investment results, for which reason we consider only one basic type of plan structure.

2.1 Modeling the Basic Structure of the DC Plans

The basic structure of the underlying DC plans is similar in all plans. They are characterized by the following specifics:

- A contribution period or savings phase of 40 years. To capture a practical and realistic situation, we assume that an employee enters the plan at age 25 and makes regular payments to it until he or she reaches retirement at age 65.
- During this active working period, we assume regular contributions, which are specified as a fraction of the current salary, are being made. To make our model clearer and easier to understand, we assume a contribution rate of 10% of the current salary.
- To model a realistic salary process, career and productivity aspects include patterns of typical wage increases over time. These two factors depend on the age of the saver and their modeling is based on the Panel Study of Income Dynamics[2] (PSID polynomial) – as shown in Figure 1.

Figure 1: Illustration of the wage dynamics with a fixed inflation rate



The wage dynamics during the savings period are based on the following assumptions:

[2] J. Cocco, F. Gomes, and P. Maenhout: Consumption and Portfolio Choice over the Life-cycle. Review of Financial Studies, Vol. 18 (2005).

The initial salary of 10’000 currency units, and its increase is modelled as:

$$wage_t = \begin{cases} 10'000 & t = t_0 = 25 \\ \text{wage}_{t-1} \times \left(1 + \text{inflation}_t\right) \times \text{career/pro ductivity factor}_t, & \forall\, 25 < t < 65 \end{cases}$$

The modeling of an average career (assumed to be independent of financial market developments) and of a salary development that is stochastically influenced only by inflation does not distort the results. With the focus on default investment options, we are interested in any uncertainty stemming from the investment side and not from other design elements of the DC plan. The probability distribution of the last salary before retirement (i.e. wage_{64}) can be seen in Figure 8.

The contributions are invested in an asset portfolio, which is managed according to the plan’s investment management strategy. These strategies are introduced in the subsequent Section 3. It is assumed that an employee does not end the plan during the savings phase (neither by accident nor due to death or disability) and that he does not withdraw funds during the accumulation phase. Market shocks or discontinuity in the wage profile, due to unemployment for instance, are certainly a realistic feature. However, considering such aspects will not contribute further insights to the intended analysis and is therefore not part of this study.

The savings phase ends at the age of retirement, which is assumed to be 65 in our modeling. The subsequent decumulation phase is characterized by the plan’s payout option.

2.2 Modeling the Retirement Phase

The retirement phase is characterized by the level of retirement income, its distribution over time and its duration – the latter of which, longevity, is quantitatively captured by mortality tables. Again, to be able to compare the results, mortality rates and resulting life expectancies are taken from the same standard mortality table – in this case, the tables for occupational pensions in Germany (RT Heubeck 2005 G)[3].

The structural distribution of retirement income depends on the payout option of the plan. Referring to a field study undertaken by the OECD[4], this study considers 5 different designs of payout options. They often go along with different regulatory frameworks.

[3] K. Heubeck, R. Herrmann, and G. D’Souza (2006), “Die Richttafeln 2005 G – Modell, Herleitung, Formeln”, Blätter der DGVFM, XXVII (3). Further research may consider different moratality tables used in different contries and assess their impact of this model component.

[4] Antolin, P. (2008), “Policy Options for the Payout Phase”, OECD Working Papers on Insurance and Private Pensions, No. 25.

The aim of this study is not to compare different payout options. Each analysis of investment strategies assumes a given payout option (PO) design.

PO1 Constant life annuity:

A life-long annuity is purchased at age 65 in which the lump sum accumulated up to this age is used as the single premium for an immediate life annuity. The benefit payments are considered being constant until death. The calculation of the annuity takes into account the term structure of the interest rates at age 65 and the standard mortality table[5]. No death benefit is paid.

$$\textit{Payments} = \textit{Lump Sum} \,/\, \sum_{t=65}^{115} {}_{t-65}p_{65} \cdot \left(1 + {}_{t-65}r_{65}\right)^{-t+65}$$

where ${}_{t}p_{65}$ is the probability that a person aged 65 stays alive for at least t years, ${}_{t}r_{65}$ is the term structure of interest rates at age 65. Payments means survival benefit payments. Note: Age 115 is the end of the used mortality table.

PO2 Inflation-linked life annuity:

Similar to the first payout option, a lifelong annuity is purchased at age 65. In this case, the payments are periodically increasing with the actual inflation. The calculation of the annuity takes into account the term structure of the interest rates at age 65, the term structure of inflation rates at age 65 and the standard mortality table. No death benefit is paid.

$$\textit{Payment at age 65} = \textit{Lump Sum} \,/\, \sum_{t=65}^{115} {}_{t-65}p_{65} \cdot \left(1 + {}_{t-65}i_{65}\right)^{t-65} \cdot \left(1 + {}_{t-65}r_{65}\right)^{-t+65}$$

$\textit{Payment}_t = \textit{Payment}_{t-1} \times (1+\textit{inflation}_t)$, for $t > 65$.

where ${}_{t}p_{65}$ is the probability that a person aged 65 stays alive for at least t years, ${}_{k}r_{65}$ is the term structure of interest rates at age 65, and ${}_{k}i_{65}$ is the term structure of inflation rates at age 65, inflation is the actual inflation and Payment means survival benefits payment. Note: 115 is the end of the used mortality table.

Note: In case of annuities, the retiree has no control over the accumulated money any more as an annuity is bought. Therefore no investment strategy has to be defined in the retirement phase.

[5] For the pricing of the annuity one could consider various cost factors incurring when such an annuity is bought over the market. The consideration of such factors would affect only the level of retirement income but not change the assessment of the different investment strategies.

PO3 Withdrawal program with fixed payments:

Under this payout option the financial assets accumulated during the savings period continue to stay in a personal account of the retiree. The payments are scheduled for a fixed period which is as long as the life expectancy, i.e. from the age of 65 to the age of 89[6]. The calculation of such an "annuity certain" takes the expected investment return for the years 65 to 89 into account – thus, the higher the expected return of the underlying investment strategy, the higher the fixed payment during the payout phase. The withdrawal rate is calculated only once at the beginning of the retirement period. In case assets are exhausted before the retiree reaches the age of 89, there are no benefit payments. In case any assets are left over after that age, the value is paid as a lump sum at the age of 89.

$$\textit{Payments} = \textit{Lump sum} \,/ \sum_{t=65}^{65+LE(65)} \left(1+\mathrm{expReturn}_t\right)^{-t+65}$$

where *LE(65)* is the life expectancy at retirement age of *65* (which is *24* years) and *Payments* means survival benefits payment.

PO4 Withdrawal program with variable payments:

Under this payout option the financial assets accumulated during the savings period continue to stay in a personal account of the retiree and lifelong retirement income will be generated. The amount is calculated by dividing the current value of the assets by the current life expectancy, thus the withdrawal rate is recalculated each year. In case of death, the current value of assets is bequeathed to the person's heirs.

$$\textit{Payment}_t = \textit{Portfolio Value}_t \,/\, LE(t), \text{ for } t > 64$$

where *LE(t)* is the life expectancy at age *t.*

PO5 Withdrawal program with variable payments supplemented by a deferred inflation-linked life annuity:

This payout option 5 consists of a combination of PO 2 and PO 4. It defines a withdrawal payment and a deferred inflation-linked life annuity that starts the benefit payment at the age of 80. The premium used to purchase a deferred inflation-linked life annuity is calculated so that the first annuity payment is similar to the last withdrawal payment at

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[6] Note: The person starting the DC plan today is assumed to be 25 years old. Based on the underlying generation mortality table, the life expectancy of this person when he reaches the age of 65 (in 40 years from now) will be 24.

age 79 (taking into account that the strategy depends on the expected asset return for the period between the age of 65 and 79).

The payments of the withdrawal plan are calculated by dividing the current amount of assets by the remaining deferral period. At age 65 the benefit payment equals the lump sum minus the premium used to purchase the deferred inflation-linked life annuity divided by 15. In case of death before the age of 80, the current value of assets is bequeathed to the person's heirs and no life annuity is being paid.

2.3 Including Mortality in our Analysis

The impact of mortality is an important component to be considered in such an analysis. In order to include mortality during the payout phase, different cash flow variables are calculated during that period allowing for survival and death probabilities. Given a specific payout option, the following cash flow is calculated in each year t of the savings and retirement phase:

$$cashflow_t = \begin{cases} Contribution_t & \text{for} \quad 25 \le t \le 64 \\ -\,Payment_t & \text{for} \quad t = 65 \ \text{ (i.e. 1st payment)} \\ -\ {}_{t-65}p_{65} \cdot Payment_t \ -\ {}_{t-65-1}p_{65} \cdot q_{t-1} \cdot Assets_{t-1} & \text{for} \quad 65 < t \le 114 \\ -\ {}_{t}p_{65} \cdot Payment_t & \text{for} \quad t = 115 \ \text{ (i.e. last payment)} \end{cases}$$

where q_t is the probability that a person aged t dies within one year, which is used to calculate the amount of bequeathed to person's heirs. ${}_kp_{65}$ is the probability that a person aged 65 stays alive for at least k years. Both probabilities are taken from the standard mortality table. Payments means paying the survival benefits payment. Note: 115 is the end of the used mortality table.

This variable measures all cash flows during the entire simulation horizon including both the savings and the payout phase. Thus, this variable is equal to the contributions during the accumulation phase ($25 \le t < 65$) and it is equal to the benefit payments (adjusted by the survival and mortality probabilities) during the decumulation or payout phase ($65 \le t < 116$).

3 Modeling Investment Strategies

This study focuses on investment strategies and on the question whether different strategy designs can have a significant effect on retirement income from DC pension plans. On the basis of the results found, we would like to discuss whether certain strategies are more adequate as default options than others and if so why.

Based on a field study conducted by the OECD, a broad set of investment strategies was selected with a clear focus on life-cycle investment strategies that had different designs in their exposure to risky assets. We use a total of 14 different strategic asset allocations (SAA) in which the portfolio allocation depends exclusively on the age of the shareholder.

In order to be able to draw useful comparisons, we included additional strategies as well. These comprise: 4 fixed portfolio investment strategies with a portfolio allocation that remains constant over time. In addition, we study 5 dynamic (path dependent) investment strategies in which the portfolio allocation does not only depend on the remaining time before a specified target date is reached but also on the current amount of assets as well as on the current capital market conditions.

To simulate the results of an investment strategy over a multi-period investment horizon, we need to specify the available asset universe and the stochastic characteristics of the financial instruments and portfolios. Clear rules on how the investment portfolio is composed based on the available financial instruments have to be established, too. In order to achieve this, we also need to find out how and when adjustments of the portfolio weights are made. This information is summarized as an investment strategy.

3.1 Asset Universe and the Stochastics of Financial Instruments

The asset class universe defines the basic types of financial instruments in which the contributions of the DC plan can be invested. This analysis focuses on the main asset classes that represent the traditional spectrum most pension plans invest their money in. The universe considered in this study comprises the following main asset classes:[7]

- Equity: We always consider a broadly diversified portfolio of international stocks like an index portfolio on the MSCI world equity index.
- Government bonds: We always consider a broad portfolio of government bonds with different maturities. Portfolio duration is 5.6 years.
- Inflation-linked bonds: We always consider a broad portfolio of inflation-linked government bonds with different maturities. Portfolio duration is 5.6 years.
- Cash: This is represented as a portfolio of money market positions.

[7] Note: Not explicitly considered are alternative asset classes such as real estate, commodities, private equity or hedge funds nor specialized investment such as infrastructure. Investments in these asset classes will be found in many real DC investment plans. For the sake of simplicity, however, they will not be included in this analysis. On the bond side, we focus on government bonds to assume investments with highest security only.

In the following discussions the risk of a portfolio is often associated with its exposure to equity positions.

Next, we look at the stochastic behavior of underlying asset classes. The projection of asset portfolios over a multi-period investment horizon requires clear quantitative modeling which expresses possible future periodical returns. For our analysis we use financial market scenarios for all asset classes to model this behavior which are produced by the risklab Economic Scenario Generator (ESG)[8].

This conceptual framework incorporates fundamental macroeconomic factors used to describe the evolution of interest rates and equities. Using a cascading structure, it captures the long term economic relationship between e.g. interest rates and inflation while allowing for short term deviations. This setting allows for an integrated modeling of financial markets, delivering economically meaningful and consistent scenarios. In order to build up a framework, 10'000 scenarios are modeled to represent the investment return of each asset class used in this study.

Calibrating the stochastic processes requires the input of a set of parameters. Specific assumptions like long term expectations and volatility are used to model macroeconomic variables or the term structure of the interest rates (used to model government bonds) and the term structure of the inflation rates (used to model inflation-linked bonds) among others. Specific assumptions are also used to model other asset classes. We derive forward looking assumptions from historical data and set the mean reversion levels of interest rates and inflation with possible fluctuations. The scenarios used in this study reflect a general economy and do not represent a specific country.

A basic characterization of the asset classes is illustrated by the parameter assumptions on return and volatility (implicitly) made for the different asset classes (see Table 1):[9]

Table 1: Risk and return characterization of underlying asset classes (represented as respective index portfolios)

	Return	Volatility
Equity (*)	7.5%	20.0%
Government bonds (**)	4.8%	3.0%
Inflation linked bonds (**)	4.5%	1.9%
Cash	3.7%	1.7%

(*) based on the MSCI World Index.
(**) portfolio duration of 5.6 years.

[8] For details see Appendix 7.1.

3.2 Investment Strategies

Investment strategies describe how asset classes are being composed to build a portfolio and how this composition is adjusted over time. Long term target date funds with an underlying life-cycle strategy are widely found and advertised in pension finances. Life-cycle concepts found in real life vary quite significantly. The analysis will focus on such strategies and will draw a comparison of special glide path designs to assess which of them are suitable as default options. To supplement this analysis we also include "fixed strategies" and "dynamic strategies".

In order to maintain their basic characteristics, the portfolios need to be rebalanced periodically. Under the simulation this is done every three months. And moreover, if due to pre-determined weight changes (like along the equity glide path) some portfolio weights are adjusted, we assume that the reallocations for the remaning portfolio components are done in a way to ensure mean-variance efficiency of the resulting portfolio to avoid systematic biases of the results.

Figure 2: Deterministic life-cycle investment strategies with a pre-set glide path[10]

Figure 2a: Life-Cycle with Linear Decrease Glide Path

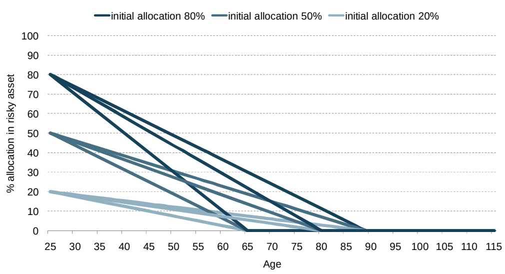


Figure 2b: Life-Cycle with Piecewise Linear Glide Path

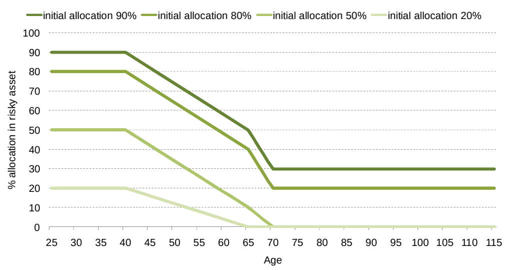


Figure 2c: Life-Cycle with Step Function Glide Path

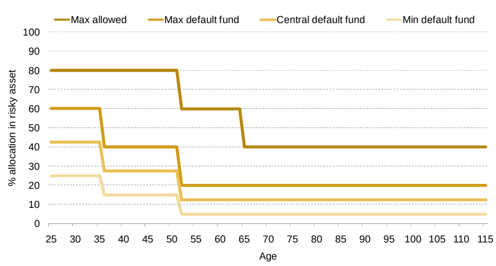
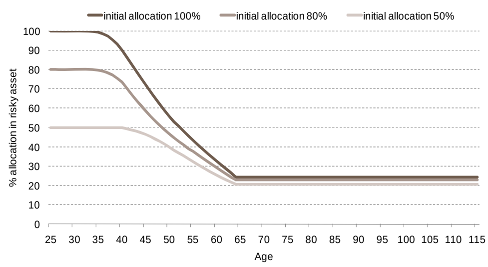


Figure 2d: Life-Cycle with Average Multi Shape Glide Path



[9] The short term volatility of the inflation-linked bonds is lower than the short term volatility of the government bonds, whereas the long term volatility of the inflation-linked bonds is higher than the one of the government bonds. This is mainly due to the volatility of the cumulatively realized inflation.

[10] The glide-path of a life-cycle strategy describes its exposure to risky investments (expresses as the portfolio weight in equity positions) over time. This exposure is typically a function of age.

- **Life-Cycle Strategy with Linear Decrease Glide Path:** The underlying rule to decrease risk exposure towards the end of the savings phase is driven by a linear function as shown in Figure 2a. The equity exposure of the asset allocation decreases over time in a linear fashion to reach an investment that carries no risk when annuity payment begins. For payout-options that imply withdrawal payments, the decrease continues until the retiree reaches the age of 89 (PO3, PO4) and age 80 (PO5), respectively.

  The initial exposure is also a parameter that is flexile and needs to be set. To capture the possible impact of this aspect, we consider three versions of a life-cycle investment strategy: LD80, LD50 and LD20 according to their starting allocations in equity with 80%, 50%, and 20%, respectively.

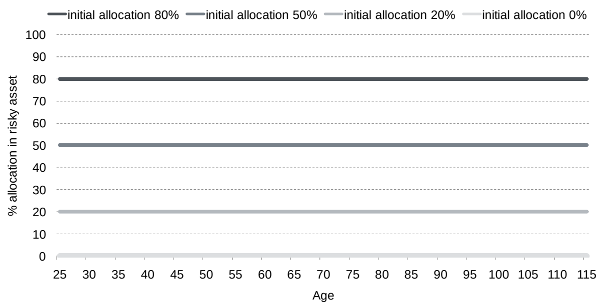
- **Life-Cycle Strategy with Piecewise Linear Glide Path:** The second type represents strategies as shown in Figure 2b, where the equity allocation dynamics are driven by a combination of several linear functions. This strategy aims to come close to the target date approach observed for example in the U.S. There are 4 life-cycle investment strategies of this kind. They are referred to as PLF90, PLF80, PLF50 and PLF20 according to their starting allocations in equity.

- **Life-Cycle Strategy with Step Function Glide Path:** The third category consists of strategies as shown in Figure 2c, where the equity allocation dynamics are driven by a step function. It aims to come close to the bracket approach observed for example in Chile. However, differing from the Chilean approach, our analysis defines the equity exposure by a step function depending on age. There are 4 versions of this kind of life-cycle investment strategy. They are referred to as SF80, SF60, SF42.5 and SF25 according to their starting allocations in equity.

- **Life-Cycle Strategy with Average Multi Shape Function Glide Path:** This fourth type of strategy is shown in Figure 2d, where the equity allocation dynamics are driven by a multi-shaped life-cycle approach. It also aims at representing an example of the target date approach observed in the U.S. It is a form derived from the dynamic multi shaped investment strategy DMS[11]. There are 3 life-cycle investment strategies of this kind. They are referred to as AMS100, AMS80 and AMS50 according to their starting allocations in equity.

[11] This strategy is explained later in this Section.

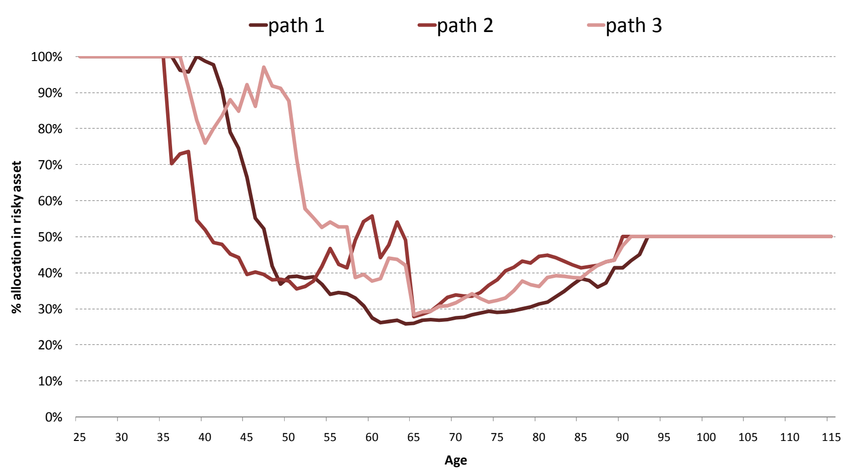
Figure 3: Consideration of Fixed Asset Allocations (life-cycle with constant glide path)



- **Fixed Asset Allocation Strategies** are characterized by a simple rebalancing strategy, where the asset allocation is kept constant over time i.e. is readjusted at every reset point in time to the starting allocation. Depending on their initial allocations to equity, we consider four versions of this kind of investment strategy (cf. Figure 3). They are referred to as FP80, FP50, FP20 and FP0 according to their (initial and ongoing) allocations in equity. The FP0 is the strategy using a pure fixed income portfolio without investments in equity.

In addition to investment strategies with pre-set asset allocations, we analyzed dynamic and path dependent asset allocation strategies. These strategies require a more complex adjustment logic. They take for example the realized performances along the individual scenario path into account to decide on the adjustment of the portfolio composition. Note that these strategies are not tactical asset allocation strategies but they are based on ex ante defined strategic rules.

Figure 4: Consideration of Life-Cycle with Multi-Shape Glide Path (illustration of three examples of possible path developments)



- **Dynamic Multi Shape Investment Strategy (DMS)**[12]**:** This life-cycle asset allocation strategy is designed to minimize the expected shortfall with respect to a pre-defined level of consumption at the end of the savings period. Based on the work of Cocco/Gomes/Maenhout[13], this level is set in this analysis at 25% of the last income. Figure 4 illustrates three samples of the glide path of the corresponding equity share. Since these paths can have multiple shapes along the timeline, the strategy is named "Dynamic Multi Shape" strategy. In each scenario, the respective performance path is applied to readjust the portfolio composition in order to avoid falling below the minimum desired level.

- **The Dynamic Risk Budgeting Investment Strategy (DRB):** Under this dynamic strategy adjustments within the portfolio asset allocation depend on the portfolio's current risk budget. The concept of this strategy is to make efficient use of the available risk budget at all times. Thus, the higher the risk budget, the more aggressive (albeit with certain restrictions) the asset allocation becomes, and vice versa.[14]

  Given our asset universe, applying risk return optimization efficient asset allocations can be derived. As illustrated in Table 2, an initial risk budget is set for each of the thirteen selected portfolios. It is defined as the maximum loss a portfolio can incur within one year. The main variable in this investment strategy is the floor. It represents the minimum portfolio value which satisfies the risk budgeting constraints. It increases with the risk free rate and with contributions and it decreases with outgoing benefit payments.

  At the beginning of each year, the floor is reset so that the current risk budget equals the initial risk budget. At each time step of a simulation, the risk budget is re-calculated given the current values of the floor and the portfolio.

$$\text{Risk Budget}_t = 1 - \frac{\text{Floor}_t}{\text{Portfolio Value}_t}$$

  A limited number of portfolios are valid within the dynamic risk budget strategy. When a portfolio *i* is simulated, only its four neighbor portfolios (i-2, …, i+2) are considered as candidates for portfolio adjustments. Thus the current risk budget will be compared in each time step with 5 initial risk budgets representing the portfolios

[12] This investment strategy represents a simplified version of a dynamic life-cycle model, developed by risklab. For more details, see Appendix 7.2.2.

[13] J. Cocco, F. Gomes, and P. Maenhout: Consumption and Portfolio Choice over the Life-cycle. Review of Financial Studies, Vol. 18 (2005).

[14] The trading frequency of such dynamic strategies is very important. Higher frequencies typically allow a more precise risk-return control but are more complex to implement. In this study, the reallocations occur on a quarterly basis.

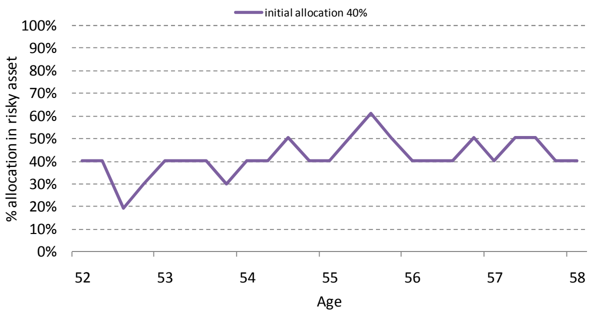
i. The portfolio that minimizes the difference between the current risk budget and its own risk budget is selected as the current/new asset allocation.[15]

There are 4 versions of this kind of investment strategy. They are referred as DRB20, DRB40, DRB60 and DRB80 according to their initial allocation in equity.

Table 2 Parameterization of risk budgets

Efficient Portfolio	1	2	3	4	5	6	7	8	9	10	11	12	13
Initial Risk Budget	12.0%	13.9%	15.5%	17.4%	19.6%	24.1%	28.6%	33.1%	37.7%	42.2%	45.6%	50.1%	54.7%
Cash	5.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Government Bond	22.7%	66.7%	87.8%	84.9%	80.4%	70.2%	59.8%	49.4%	39.0%	28.6%	20.8%	10.4%	0.0%
Inflation Linked Bond	71.5%	27.9%	2.1%	0.6%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Equity	0.8%	5.4%	10.1%	14.6%	19.5%	29.8%	40.2%	50.6%	61.0%	71.4%	79.2%	89.6%	100.0%

Figure 5: Dynamic Risk Budgeting: illustration of a glide path with an initial equity allocation of 40%



3.3 Assessing the shape of the life-cycle strategies

Considering their changing allocation behaviour over time it is not obvious how to classify some of the investigated life-cycle investment strategies in more risky or less risky types. The starting equity exposure is only a very limited and sometimes misleading indicator of a strategy’s risk-return characteristics because it does not comprise any further information of the equity exposure over the lifetime of the strategy. To capture the average equity exposure of an investment strategy, two measures are introduced: The time weighted share of equity exposure and the volume weighted exposure.

[15] For more details, see Appendix 7.2.1.

(TWS) Time Weighted Share of Equity Exposure: $$TWS = \frac{1}{\#time\ step} \times \sum_{t=25}^{IT} \omega_{equity}(t)$$

(VWS) Volume Weighted Share of Equity Exposure: $$VWS = \frac{1}{\sum_{t=25}^{IT} V(t)} \times \sum_{t=25}^{IT} \left[\omega_{equity}(t) \times V(t)\right]$$

Note: For the payout options 1 and 2, investments occur between the age of 25 until the retirement age 65, therefore the investment time is $IT_{PO1} = IT_{PO2} = 65$. Other payout options have longer investment horizons: $IT_{PO3} = 89$, $IT_{PO4} = 115$, and $IT_{PO5} = 80$. V(t) denotes the portfolio value and $\omega(t)$ the equity exposure at time t.

The two measures reflect different perspectives[16] on the "average exposure" and thus do not necessarily lead to the same assessment. However, this aspect may become relevant, when for example a regulator wants to put restrictions on the acceptable risk profile for default options strategies and associates the riskiness of the underlying DC plan with its average equity holdings. Putting a general restriction on a maximum share in equity (e.g. 30%) or a restriction on the maximum average equity exposure (e.g. also 30%) could yield completely different feasible strategies. Thus to avoid inefficient regulatory rules or plan specific bylaws it is important to have a clear understanding of what kind of risk profile is intented.

Figure 6: Comparing the shape of life-cycle glide path

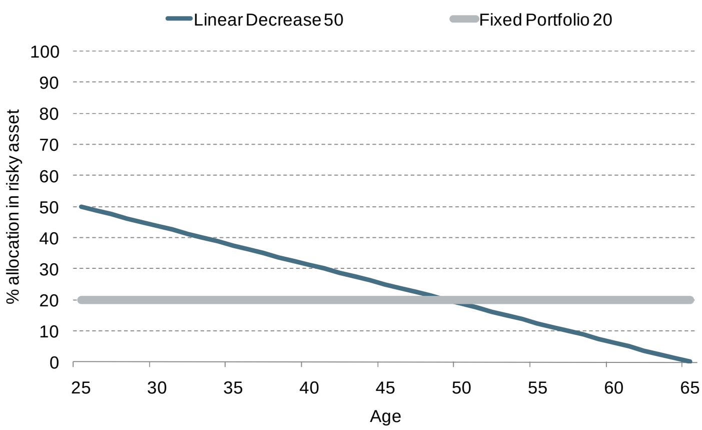


Figure 6 illustrates this issue. The two example strategies FP20 and LD50 show different glide path behaviour over time. Applying the TWS measure, these strategies would be assigned an average equity exposure of 20% and 25%, respectively, i.e. LD50 would be assessed as the strategy implying more risk. But calculating the VWS would lead to a

[16] If the value of the portfolio keeps growing during the savings phase, the equity volume during the period [45; 65] is higher than in the period [25; 44] (Portfolio Size Effect). Thus a value weighted exposure measure takes into account that the equity exposure during phases of high portfolio volumes affect the resulting lump sum stronger than the ones with lower volumes.

risk of 20% and 13.1% respectly, thus, assessing FP20 as the strategy bearing more risk.

However, aiming to assess the riskiness of the underlying DC plan the risk has to be measured in terms of replacement rate risk e.g. expressed as Value at Risk.[17] Here we will find e.g. for PO1 a replacement rate of 17.8% for FP20 and 22.6% for LD50, respectively.[18] As the lower the replacment rate the higher the risk, we can derive that FP20 implies more risk than LD50. This result correlates with the VWS as a risk measure and emphazises that TWS is not an adequate indicator for the implied riskiness of an investment strategy.

This example also illustrates that a maximal equity exposure is not appropriate either. Assuming e.g. a maximum equity exposure of 40% would define FP20 to be valid and LD50 to be invalid. Thus, the investment strategy implying less risk for the employee would not be admissible.

Table 3: Volume weighted share and time weighted whare in risky asset (equity)

	Constant Life Annuity		Inflation Linked Life Annuity		Withdrawal Program Fixed Payment		Withdrawal Program Variable Payment		Withdrawal Program + Deferred Annuity	
	TWS	VWS	TWS	VWS	TWS	VWS	TWS	VWS	TWS	VWS
FP 0	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
FP 20	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%	20.0%
FP 50	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%	50.0%
FP 80	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%	80.0%
LD 20	10.0%	5.3%	10.0%	5.3%	10.0%	7.0%	7.1%	6.0%	10.0%	7.4%
LD 50	25.0%	13.1%	25.0%	13.1%	25.0%	17.7%	17.9%	14.9%	25.0%	18.4%
LD 80	40.0%	20.8%	40.0%	20.8%	40.0%	28.2%	28.6%	23.7%	40.0%	29.3%
SF 25	14.3%	8.3%	14.3%	8.3%	10.8%	6.4%	9.2%	6.1%	11.8%	7.2%
SF 42.5	26.4%	17.4%	26.4%	17.4%	21.3%	14.6%	18.8%	14.1%	22.7%	15.8%
SF 60	38.5%	26.5%	38.5%	26.5%	31.7%	22.7%	28.4%	22.1%	33.6%	24.3%
SF 80	72.7%	65.5%	72.7%	65.5%	60.6%	50.4%	54.7%	47.8%	63.9%	56.7%
PLF 20	13.7%	8.2%	13.7%	8.2%	8.6%	3.5%	6.2%	2.8%	10.0%	5.5%
PLF 50	37.3%	26.1%	37.3%	26.1%	23.8%	12.1%	17.0%	9.7%	27.7%	18.4%
PLF 80	67.3%	55.7%	67.3%	55.7%	50.5%	36.8%	41.8%	33.0%	55.4%	45.5%
PLF 90	77.3%	65.5%	77.3%	65.5%	60.5%	46.5%	51.8%	42.6%	65.4%	55.3%
AMS 50	41.0%	32.3%	41.0%	32.3%	33.4%	25.5%	29.8%	24.4%	35.5%	28.3%
AMS 80	56.9%	39.3%	56.9%	39.3%	44.3%	29.8%	38.1%	28.1%	47.7%	33.8%
AMS 100	69.3%	45.9%	69.3%	45.9%	52.6%	33.4%	44.5%	31.2%	57.2%	38.6%
DRB 20	20.9%	21.6%	20.9%	21.6%	21.5%	21.5%	21.3%	21.5%	20.7%	21.6%
DRB 40	40.5%	42.1%	40.5%	42.1%	41.6%	42.0%	41.0%	42.0%	40.0%	42.0%
DRB 60	59.7%	62.3%	59.7%	62.3%	61.4%	62.3%	60.5%	62.3%	59.0%	62.3%
DRB 80	78.0%	81.5%	78.0%	81.5%	80.3%	81.5%	79.0%	81.5%	77.0%	81.5%
DMS	61.3%	38.1%	61.3%	38.1%	50.1%	31.9%	49.3%	32.3%	52.0%	33.7%

[17] Regarding adequate risk measures cf. Section 4.2.1.

[18] For these numerical results see Appendix 7.3.

The example underscores that the choice of the measure used to assess the shape of the equity glide path (and with that provide a proxy for the riskiness of a strategy) can be crucial and must be designed carefully to avoid inefficient regulation. *Table 3* summarizes the TWS and the VWS in equity of the 23 investment strategies for the 5 payout options illustrating also the different effects induced by the structure of the payout option.

4 Modeling Retirement Income from DC Plans

From the perspective of an employee or retiree, DC pension plans bear significant investment risk, i.e. the amount and the duration of future retirement income derived from this type of pension plan is highly uncertain. The projection results clearly show, for a retiree there are chances to achieve a high pension income but also risks of ending with low income, and under some plans, there is even the danger of exhausting the fund completely and being left without any retirement income at all. Furthermore, this raises the question of how much risk an employee is willing to take for a possible pension above average.

To provide a quantitative assessment of the risk-return characteristics of DC pension retirement income we analyze potential outcomes based on 10’000 long-term scenario projections (comprising a time span from age 25 until 115 years), which simulate the accumulation and decumulation process of a specific DC plan. The calculations of potential retirement income take into account the stochastic projections of pensionable salary (determining contributions) as well as the given payout option in combination with the chosen investment strategy (generating stochastic returns and payouts) over the considered time horizon.

4.1 Replacement Rate Distribution

The level of retirement income during the decumulation phase is often measured in relation to the last income before retirement. This relation defines the replacement rate achieved by the plan and gives the retiree a good orientation – a perspective that might also be relevant for a regulator or government agency when assessing the suitability of a default investment option.

Based on the different scenarios, we can derive a corresponding empirical distribution of replacement rates for each of the different DC plans. Since the retirement income will vary over time, the relation to the last wage level will change just as well. In order to be able to compare the expected income at different stages in a retiree’s lifetime, we discount payments[19] to the age of retirement while using the government bond rate of return (GBR): $\forall s \in \{1,...,\#\text{scenarios}\}$,

$$\text{RepRate}_s = \frac{1}{LE(65)} \cdot \sum_{t=65}^{115} \left[{}_{t-65}p_{65} \cdot \prod_{l=0}^{t-66} \frac{1}{(1+\text{GBR}_{65+l}^s)} \cdot \frac{\text{Payment}_t^s}{\text{Wage}_{64}^s} \right],$$

where

Payment_t^s is the retirement income in this scenario *s* from the DC plan at age *t*.

Wage_{64}^s is the last wage level of the employee at the time before retirement.

GBR_{65+l}^s describes the return of the government bond portfolio in scenario *s* in year *l* after retirement which is used to discount future income to the time of retirement.

${}_k p_{65}$ is the probability that a person aged 65 stays alive for at least *k* years.

$LE(65) = \sum_{t=0}^{115-65} {}_t p_{65}$ is the life expectancy at retirement age of 65 (which is 24 years). Thus the factor $\frac{1}{LE(65)}$ is used to obtain a value which is normalized in terms of the last wage.

4.2 Defining Risk and Return Measures for DC Plans

The risk and return measures described in the following Section will be based on corresponding replacement rates distributions:

4.2.1 Measures of Risk for Retirement Income

One of the main concerns of retirees is that their retirement income might be very low or even below a critical level. There are different ways of assessing the downside potential of a replacement rate. The following measures of risk deal with this issue from different points of view:

- ($\text{VaR}_{5\%}$) Value at Risk of the replacement rate distribution on a 95% confidence level:

  This risk-measure describes the result that could happen under very unfavorable circumstances. The measure represents the highest replacement rate value achieved by the 500 worst scenarios. Thus, in 95% of the scenarios, the

[19] In case RepRate is calculated without allowing for discounting, the results as shown in Section 5 are basically the same.

replacement rate values are higher than this risk level. This risk-measure is directly computed by identifying the 5% percentile value of the emperical replacement rate distribution.

$$\mathrm{VaR}_{5\%} = \inf\{x, \mathrm{P}(\mathrm{RepRate} < x) \geq 5\%\},$$

where RepRate stands for replacement rate.

Note that the higher the Value at Risk the lower the risk as replacement rates are analyzed here.

- ($\mathrm{CVaR}_{5\%}$) Conditional Value at Risk of the replacement rate distribution on a 95% confidence level (which gives the expected replacement rate in the 5% worst cases): Given the definition of the emperical replacement rate distribution, a high CVaR(5%) is better than a lower CVaR(5%).

  $$\mathrm{CVaR}_{5\%} = E[\mathrm{RepRate} \mid \mathrm{RepRate} < x], \text{ where } x \text{ is given by } \mathrm{VaR}_{5\%}$$

  Note: $\mathrm{CVaR}_{5\%} \leq \mathrm{VaR}_{5\%}$

- **Shortfall Probability on Capital Protection:** Other risk-measures can provide further information on the inherent risk of a pension plan. The probability that pension income from the DC plan will be even less than the amount of contributions paid in during the savings phase is a well used risk-measure, for example. In some countries, this capital protection – either in nominal or even in real terms – is in some countries considered to be a minimum requirement a DC plan has to fulfill to qualify as a pension plan. To calculate the probability of this kind of shortfall we use the cash flow measure introduced in Section 2.2. and the following indicator function:[20]

  $$\forall s \in \{1,...,\#\mathrm{scenarios}\}$$

  $$I(s) = \begin{cases} 1 \text{ if } \displaystyle\sum_{t=65}^{115} \left[\frac{cashflow_t^s}{\prod_{l=65}^{t-1} \left(1 + Inflation_{l-1}^s\right)} \right] < \sum_{t=25}^{64} \left[cashflow_t^s \cdot \prod_{l=t}^{64} \left(1 + Inflation_l^s\right) \right] \\ 0 \text{ else} \end{cases},$$

  $$\text{Shortfall Probability} = \frac{1}{\#\mathrm{scenarios}} \cdot \sum_{s=1}^{\#\mathrm{scenarios}} I(s)$$

A Note on Conversion Risk: As conversion risk we define the risk that at the point of time when converting the lump sum into an annuity, capital market condition might be unfavourable. A provider of a product including accumulaton and annuity payment phase

[20] The following analyses use this risk-measure in real terms only.

should consider conversion risk explicitly in its strategy by adjusting the portfolio risk exposure towards the retirement age in such a way that the mismatch risk relative to the investment position, given the lump sum available at retirement is small. In a certain way, the investigated strategies do all respect the conversion risk by reducing the equity exposure before the retirement age, except for the fixed portfolio and dynamic risk budget strategies. The latter is focused on the market performance only.

4.2.2 Measures of Return for Retirement Income

The average level of replacement rates is a major orientation for retirees to assess the quality of an offered DC plan. To characterize the average outcome of a distribution of scenario results we will use the statistics of median or mean:[21]

- **Median** of the replacement rate distribution: This measure represents an average replacement rate the retiree can expect to achieve with a DC pension plan. For the median, 50% of the scenarios are above and 50% of them are below this value.

  This return measure is directly computed by identifying the 50% percentile value of the replacement rate distribution.

  $$\text{Median} = \inf\{x, \text{P}(\text{RepRate} < x) \geq 50\%\}$$

DC pension plans, like any other investment opportunity with an investment phase and an earnings phase, can be assessed on the internal rate of return (IRR) the invested money earns over the considered investment horizon. The IRR enables us, unlike the median which looks only at decumulation phase, to combine both the savings phase and the payout phase into one single return figure.

- **(IRR)** Internal Rate of Return of retirement investments is calculated in each scenario using the cash flow measure as defined in 2.2.:

  $$\forall s \in \{1,...,\#\text{scenarios}\}, \quad 0 = \sum_{t=25}^{115} cashflow_t^s \cdot \left(1 + irr^s\right)^{25-t}$$

  In theory the equation may have more than one solution but due to Descartes rule of signs there will be one well-defined solution in each scenario. With this emperical IRR distribution, the median of the IRR is directly computed by identifying the 50% percentile value.

[21] In this analysis we focus on median. Under asysmmetric replacement rate distributions it provides the more conservative assessment of average income levels.

5 Analysis of Retirement Income Simulations

In this Section we summarize the statistical results of the simulation analysis which we performed for the various investment strategies introduced above – given the basic DC plan assumption and a payout option. Since we do not intend to compare results of different payout options but to compare results of investment strategies within a given payout option, the presentation structure follows the different payout options.

For a simplified representation of the various investment strategies within the results diagrams we will use shorthand logograms, specific colors and symbols for the different strategies. The following table summarizes these abbreviations with the corresponding strategy names.

Table 4: Overview strategies and shorthand logograms

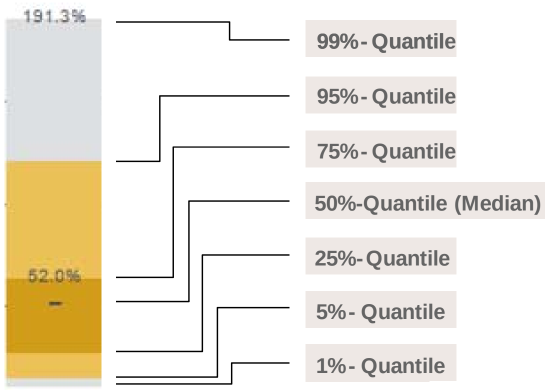
●	1	FP 0	Fixed Portfolio with 0 % start allocation in equity
●	2	FP 20	Fixed Portfolio with 20 % start allocation in equity
▲	3	FP 50	Fixed Portfolio with 50 % start allocation in equity
◆	4	FP 80	Fixed Portfolio with 80 % start allocation in equity
●	5	LD 20	Linear Decrease with 20 % start allocation in equity
▲	6	LD 50	Linear Decrease with 50 % start allocation in equity
◆	7	LD 80	Linear Decrease with 80 % start allocation in equity
○	8	SF 25	Step Function with 25% start allocation in equity
△	9	SF 42.5	Step Function with 42.5% start allocation in equity
△	10	SF 60	Step Function with 60 % start allocation in equity
◇	11	SF 80	Step Function with 80 % start allocation in equity
●	12	PLF 20	Piecewise Linear Function with 20 % start allocation in equity
▲	13	PLF 50	Piecewise Linear Function with 50 % start allocation in equity
◆	14	PLF 80	Piecewise Linear Function with 80 % start allocation in equity
■	15	PLF 90	Piecewise Linear Function with 90 % start allocation in equity
▲	16	AMS 50	Average Multi Shape with 50 % start allocation in equity
◆	17	AMS 80	Average Multi Shape with 20 % start allocation in equity
■	18	AMS 100	Average Multi Shape with 20 % start allocation in equity
●	19	DRB 20	Dynamic Risk Budget with 20 % start allocation in equity
▲	20	DRB 40	Dynamic Risk Budget with 40 % start allocation in equity
▲	21	DRB 60	Dynamic Risk Budget with 60 % start allocation in equity
◆	22	DRB 80	Dynamic Risk Budget with 80 % start allocation in equity
▲	23	DMS	Dynamic Multi Shape

For the following analyses we use risk-return diagrams. The risk dimension is plotted on the abscissa, the return dimension on the ordinate. The more to the right an investment strategy is located in the diagram the more risky it is. Both risk and return measures vary during the analyses.

In addition, we provide percentile plots of replacement rate distributions to visualize the whole risk-return profile, including also the upside potential of investment strategies. The percentile plot figures show the main percentiles of the probability distribution of replacement rates. The figure below explains the coloring used. The strategies are always in the order of their volume weighted share of risky assets, starting from the left with the investment strategy comprising the lowest volume.

In addition, a summary table including all numerical results is given in Appendix 7.3.

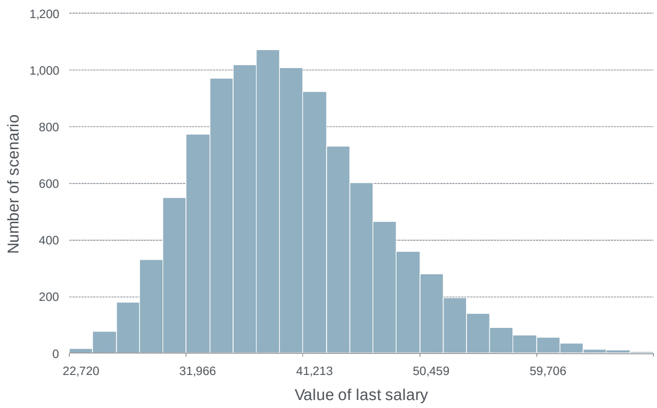
Figure 7: Explanation of percentile plots



5.1 Basic Characteristics of DC Plan

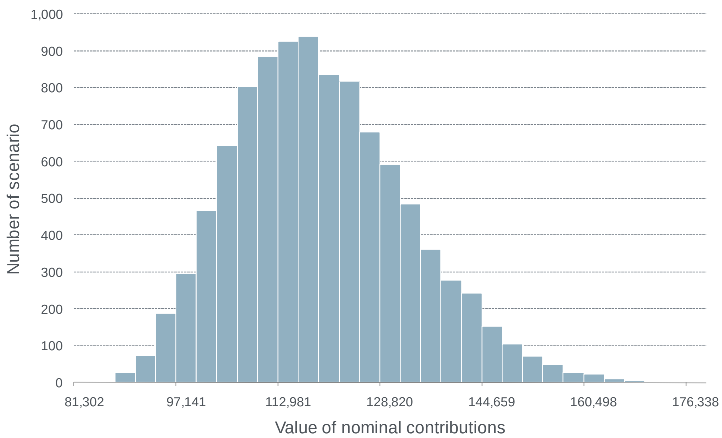
A first insight into risk-return characteristics of the underlying DC plan is provided by the analysis of potential salary developments until retirement. In Figure 8 the corresponding emperical probability distribution of the last salary before retirement is displayed. The broad range of possible results indicate that inflation risk during the savings period of 40 years has a substantial impact on final salary levels – even given a deterministic career path, which would add some further risk to the results.

Figure 8: Histogram: Distribution of last salary before retirement



Corresponding to a stochastic salary development the probability distribution of contributions paid into the savings plan can be derived. Figure 9 displays the range of sums of cash-in-flows. Under a nominal capital protection this sum will be the required minimum retirement income during the decumulation phase of the DC plan.

Figure 9 Histogram: Nominal contributions paid to savings plan

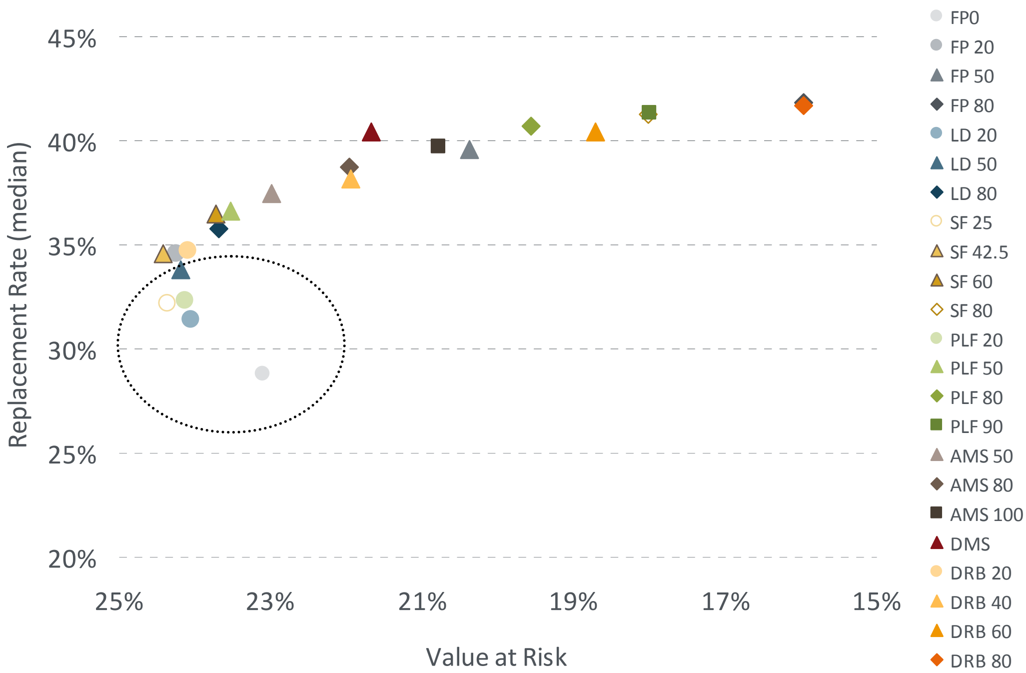


5.2 Constant Life Annuity

We start the presentation of the simulation results for the "Constant Life Annuity" payout option, being the most traditional and widely used payout option.

Figure 10 presents the risk-return profile which is based on the replacement rate, hence, the risk is the value at risk using the 5% level and the return is the median of the replacement rate distribution.

Figure 10: PO 1: risk return diagram (VaR)



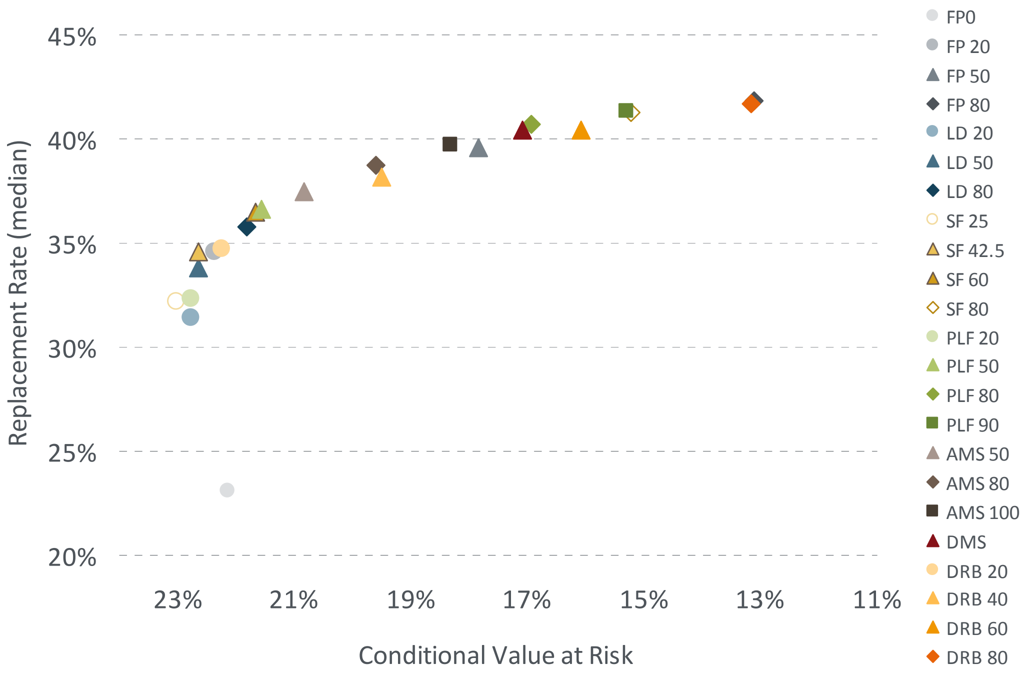
The investment strategies are located along a curve, which resembles an efficient frontier, underscoring the common view that higher return expectations go along with higher risks associated to it[22]. Some of the investment strategies appear not to be efficient given these risk- and return-measures (cf. pints in dotted circle). This holds especially for the investment strategies with low equity exposure (e.g. SF25, PLF20, LD20 and FP0) where for a given risk an investment strategy with a higher return exists. All these four investment strategies have a volume weighted share of equity exposure of aorund 8% or less.

The dynamic life-cycle strategy (DMS) is placed above the efficient frontier, hence, it has a preferable risk-return profile as it provides less risk given the same return (e.g. in comparison with DRB 60) or more return given a similar risk (e.g. in comparison with AMS 80 or DRB40).

A second view on the results under changing the risk-measure to the conditional value at risk is presented in *Figure 11*. The return measure remains unchanged.

[22] The investment strategies analyzed in this study are chosen from practical examples. Since they are not optimized with respect to a common underlying risk return paradigm, we can not expect that they will be positioned on an efficiency line. There might even exist unobserved strategies dominating others.

Figure 11: PO 1: risk return diagram (CVaR)

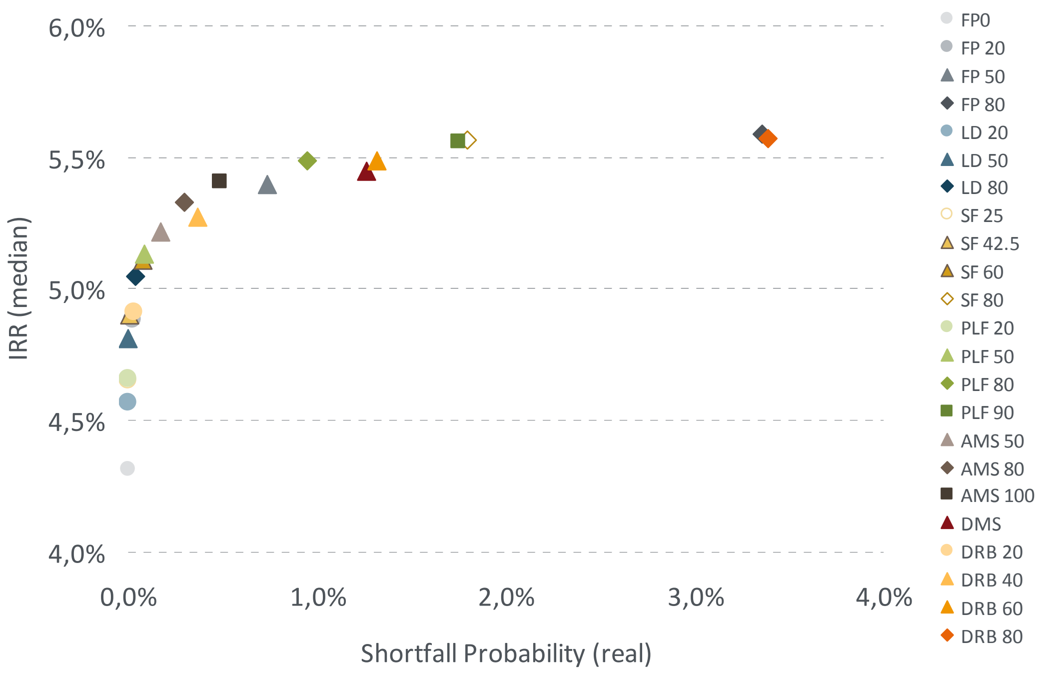


Also this figure shows that investment strategies are located along a curve, which resembles an efficient frontier. Also under this risk-measure, investment strategies with low equity exposure are not efficient. DMS is now located on the efficient frontier and does not dominate others, stressing the relevance of the relative view one takes for the assessment.

Applying a third view on the results, we now exchange the return measure as well as the risk-measure. We focus on the risk to fail capital protection in real terms, i.e. plotting the probability that benefits are less than contributions paid. Considering the DC plan as an investment process we use the internal rate of return as the return measure.

Note: It is obvious that the dominance of several portfolio strategies is more distinctive by using the Value at Risk. This could be due to the fact that the Value at Risk is focused at one specific point (the 5%-quantile) and with the Conditional Value at Risk a specific interval (0% to 5% interval) is investigated. One explaination might be the runs of the empirical distribution functions which might have some intersections in the viewed interval.

Figure 12: PO 1: risk return diagram (shortfall probability)

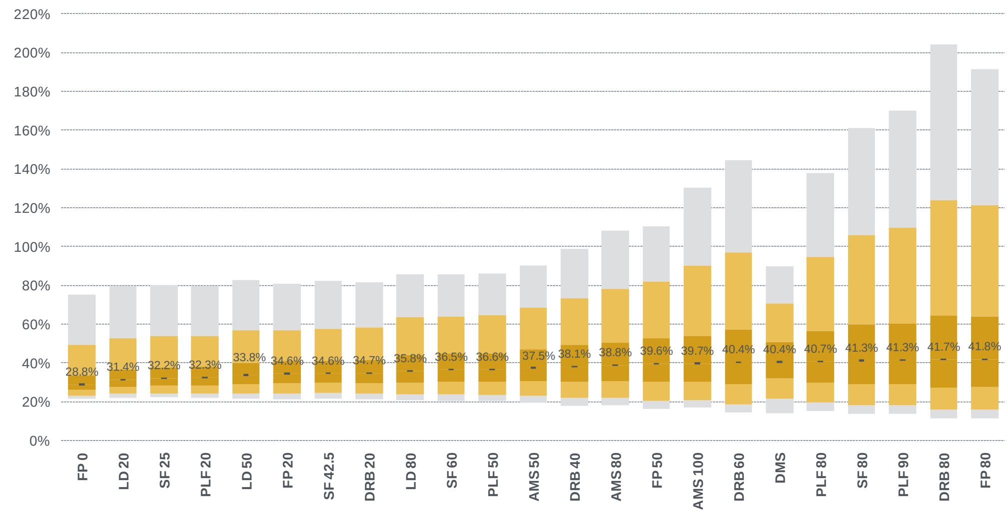


The locations of the investment strategies build again curve which resembles an efficient frontier. Many of the investment strategies have a shortfall probability below 1% and most of them below 2% which is due to the long savings phase of 40 years. Strategies with low equity exposure are inefficient in this risk-return diagram, too. In addition, also FP80 and DRB80 appear not to be very attractive as they are showing a much higher risk associated with the same return compared to e.g. PLF90. Indeed the risk in terms of having an amount of benefit payments less than the amount of contributions is very high for the DRB80 and FP80, compared to the others, which is mainly due to their high equity exposure.[23] Some strategies appear to be positioned slightly under the efficient frontier (e.g. DRB40, FP50, DMS) but differences are small.

Finally, we analyze the percentile plots of replacement rates of the investment strategies.

[23] The volume weighted share in equity is 81.5% for DRB80 is 81.5% and 80.0% for FP80 compared to e.g. 65.5% for PLF90; equity shares of the other strategies are given in *Table 3*.

Figure 13: PO 1: percentile plot of replacement rates



As *Figure 13* shows the probability distributions of replacement rates are skewed. The risk exposure is rather limited compared to the upside potential. This holds especially for investment strategies with high equity exposure. The range of replacement rates is enormous. While observing a median for the investment strategies between around 29% and 42% for some strategies we receive in 1% of the simulations replacement rates even higher than 150%.

Investment strategies with very high risk exposure also provide the highest upside potential (represented by the 75%- 95% and 99%-percentile). This holds especially for the strategies FP80 and DRB80 but which are inefficient with respect to a risk-return profile where shortfall probability and the IRR are used.

Basically, we can derive from the figure above that the higher the volume weighted equity share the higher the upside potential. Note that the strategies are in the order of their volume weighted share in risky assets starting from the left with the strategy comprised the lowest volume. However, the AMS-strategies provide a higher upside potential for a similar (or even lower) average equity share (cf. AM 80 and AMS100).

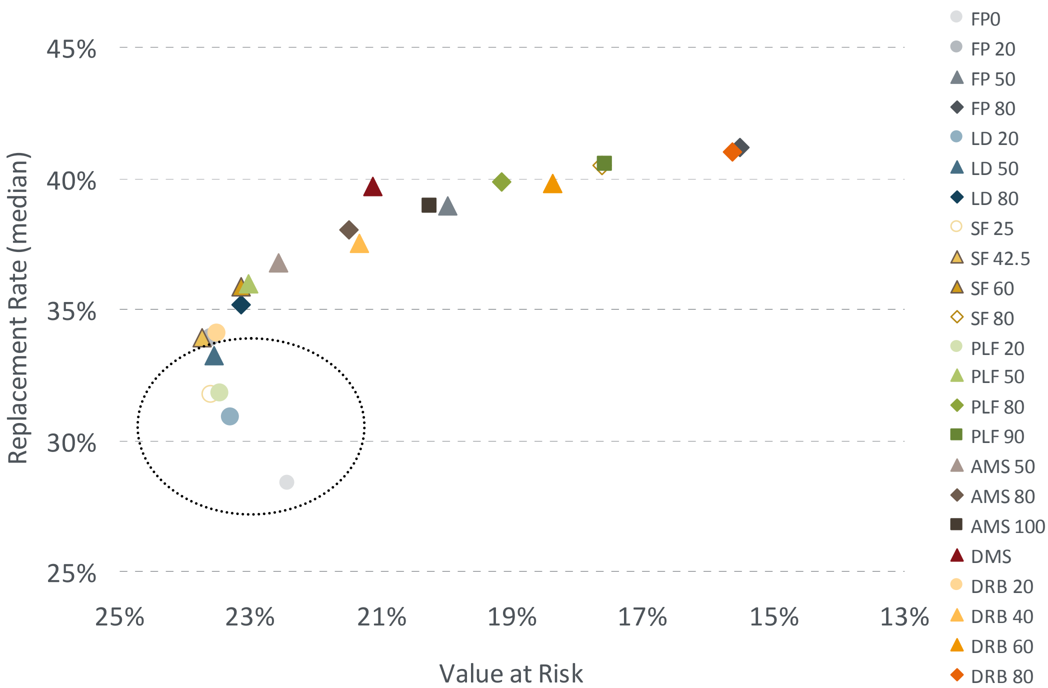
At first sight, DMS is a clear outlier in the range of the deterministic strategies and seems to be satisfying. Obviously, the volatiliy of DMS is much smaller compared to strategies with similar median (cf. also Table 5 in Appendix 7.3 showing a volatility of 16% for DMS and around 26% for DRB60 and PLF80). Having a closer look shows the benefits of such a strategy can contribute. For example, DMS has the highest 25% percentile of all strategies. This advantage comes at a cost of having a significantly lower 75% percentile compared to its neighbors. Depending on the preference of an individual member or even the regulator this could be favorable. Considering the risk of the DMS, all strategies with a higher median have a lower 5% percentile. Yet, this advantage seems not to hold for the 1% percentile.

For investment strategies which appear to be rather similar in their risk-return profile (as e.g. AMS100 and FP50) the percentile plot in *Figure 13* shows that their upside potential differs significantly. For AMS100, where the equity allocation dynamics is driven by a multi shaped life-cycle approach, the 95%- and the 99%-percentile are much higher compared to the fixed allocation strategy FP50.

5.3 Inflation-linked Life Annuity

In this Section we focus on a life-long annuity again. In contrast to Section 5.2 the annuity payment is linked to inflation. The overall results are very similar to results of the life-long constant annuity.

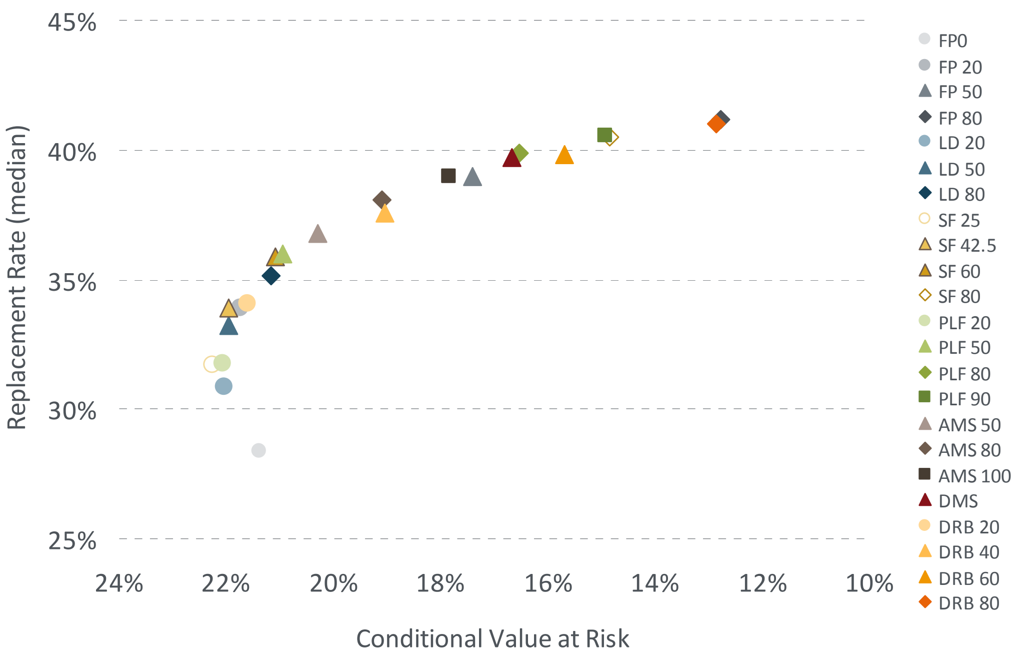
Figure 14: PO 2: risk return diagram (VaR)



The investment strategies are located along a line, which again resembles an efficient frontier. Investment strategies with low equity exposure appear not to be efficient (cf. pints in dotted circle). Also for this payout option the dynamic life-cycle strategy (DMS) has a preferable risk-return profile.

Using the risk-measure conditional value at risk in *Figure 15*, again, we obtain an efficient frontier with all strategies on it. Low equity strategies remain inefficient with this risk-return diagram.

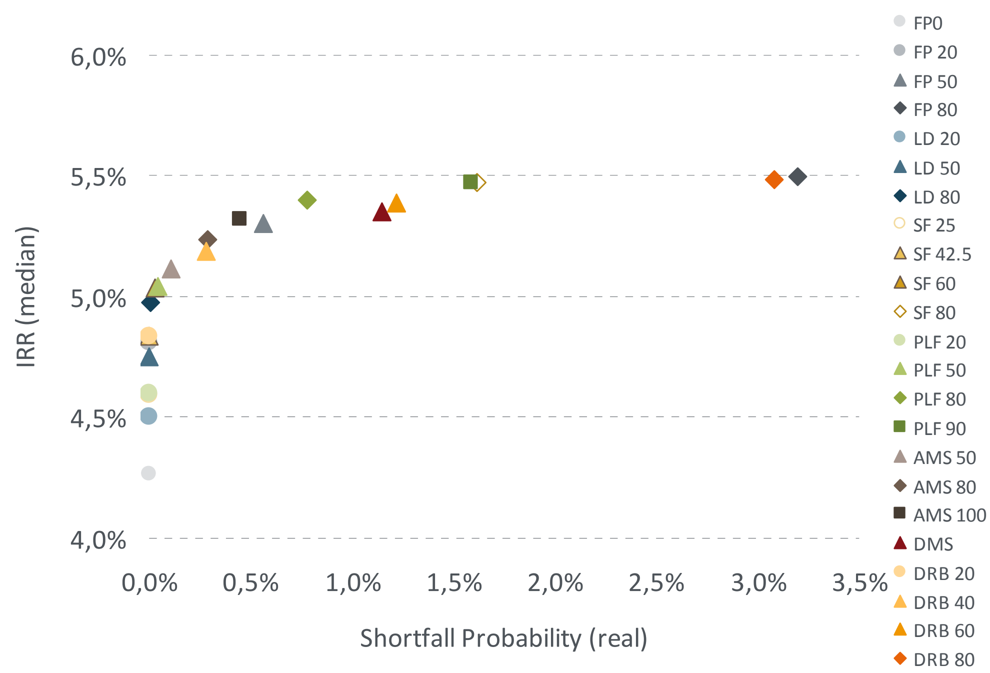
Figure 15: PO 2: risk return diagram (CVaR)



We now switch to the risk-return diagram where the risk is the shortfall that benefit payment do not exceed the contributions paid and the return is measured as the internal rate of return.

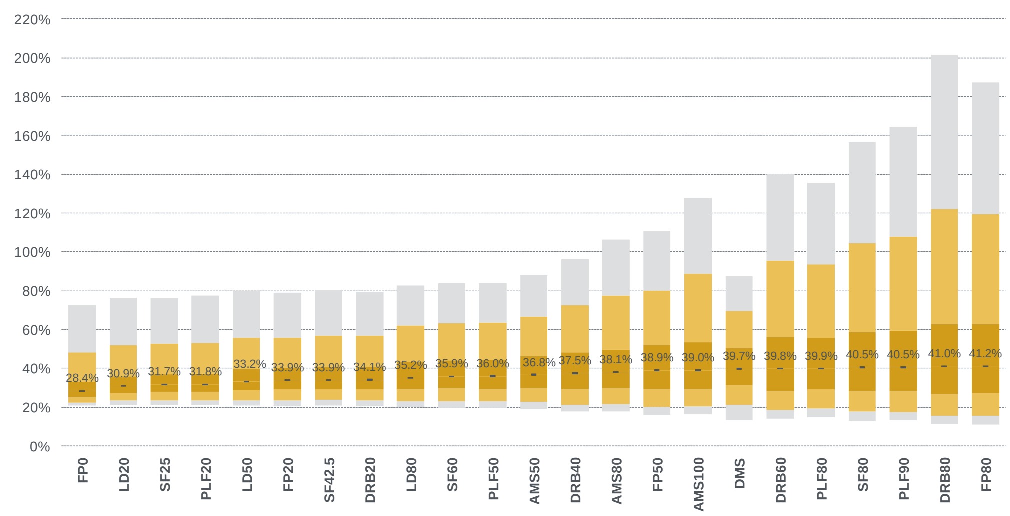
Also within this risk-return diagram the investment strategies build an efficient frontier. As for payout option 1 many investment strategies have a risk smaller than 1% and most of the strategies smaller than 2%. Low equity strategies are inefficient. Also here, some strategies are positioned slightly under the efficient frontier (DMS, DRB60) but differences are small.

Figure 16: PO 2: risk return diagram (shortfall probability)



The percentile plot of replacement rates of all investment strategies shows similar effects as described in Section 5.2.

Figure 17: PO 2: percentile plot of replacement rates

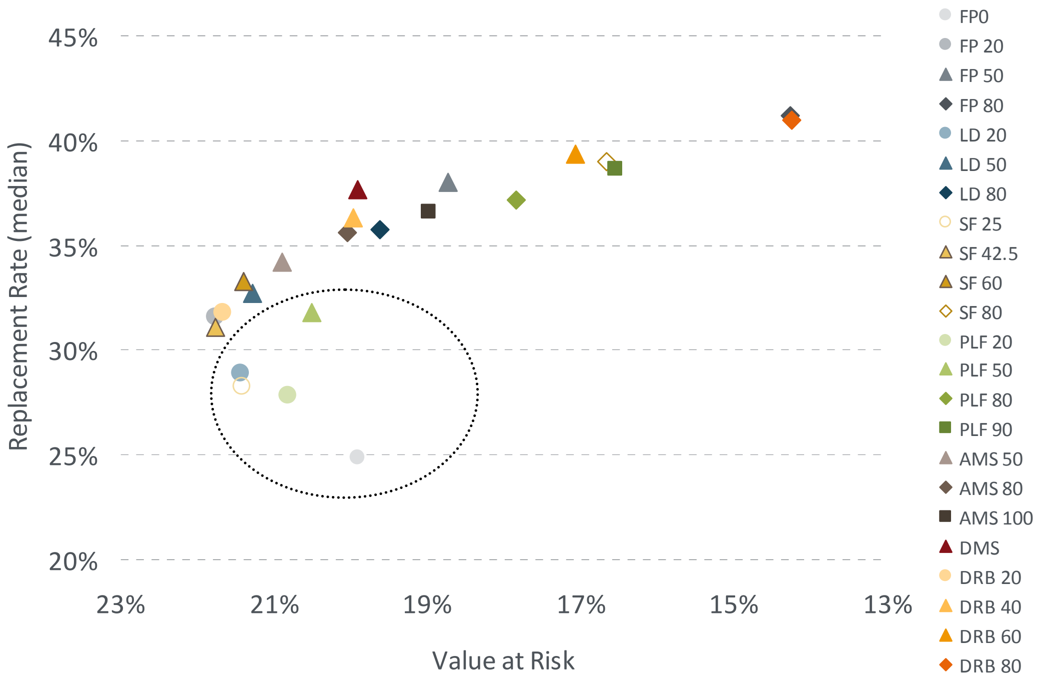


5.4 Withdrawal Program with Fixed Payment

The payout option investigated within this Section differs significantly from the two payout options analyzed above. Here, the investment strategy will affect both savings and payout phase of the pension plan. Payments are fixed until the age of 89 when benefits payment ceases.

The analysis begins with the risk-return diagram where the risk is the value at risk to its 5% percentile and the return is the median of the replacement rate distribution.

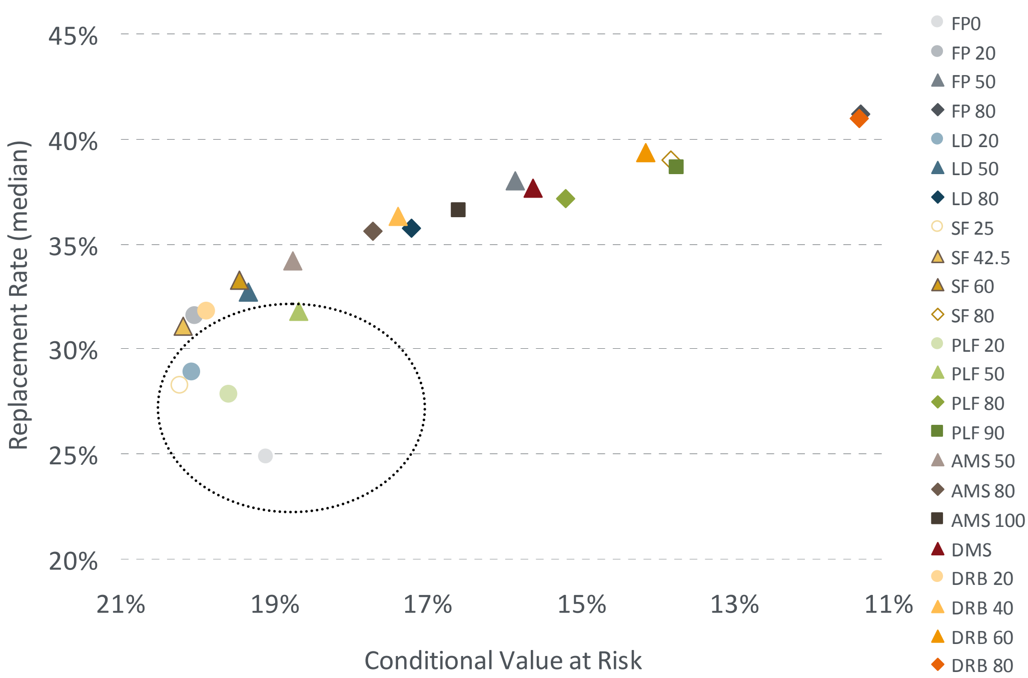
Figure 18: PO 3: risk return diagram (VaR)



As observed for the other payout options, most of the investment strategies are located along a line. Also, investment strategies with a low equity exposure are inefficient with respect to the chosen risk and return measures (cf. pints in dotted circle). As for payout options 1 and 2 these are the investment strategies with a volume weighted equity share of about 8% or less. In addition, for this payout option also all the piecewise linear function investment strategies (PLF50, PLF80 and PLF90) appear to be inefficient being located under the efficient frontier.

Again, the dynamic life-cycle strategy (DMS) has the best investment return given the associated risk. Changing the risk-measure to the conditioned value at risk eliminates this advantage (cf. *Figure 19*), here, DMS is on the efficient frontier. Moreover, investigating the percentiles in *Figure 21* we can observe its limited upside potential.

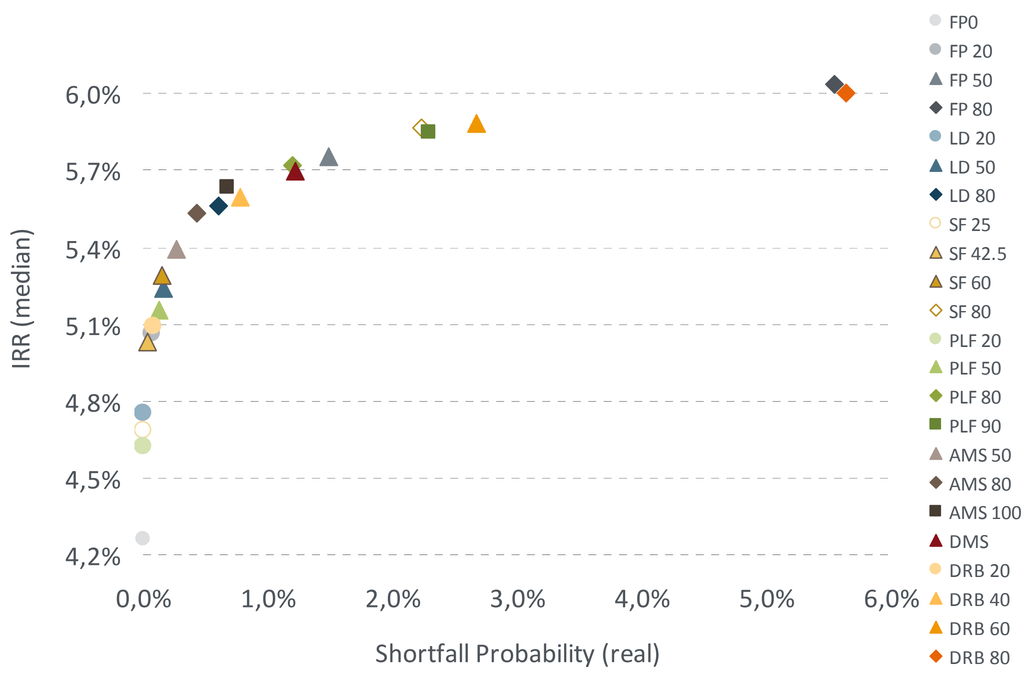
Figure 19: PO 3: risk return diagram (CVaR)



Also with the conditional value at risk on the axis of abscissae piecewise linear function life-cycle strategies appear not to be efficient. This holds especially for PLF50.

The next figure gives us the probability that contributions exceed the benefits paid. Again, we use the internal rate of return as the associated return measure.

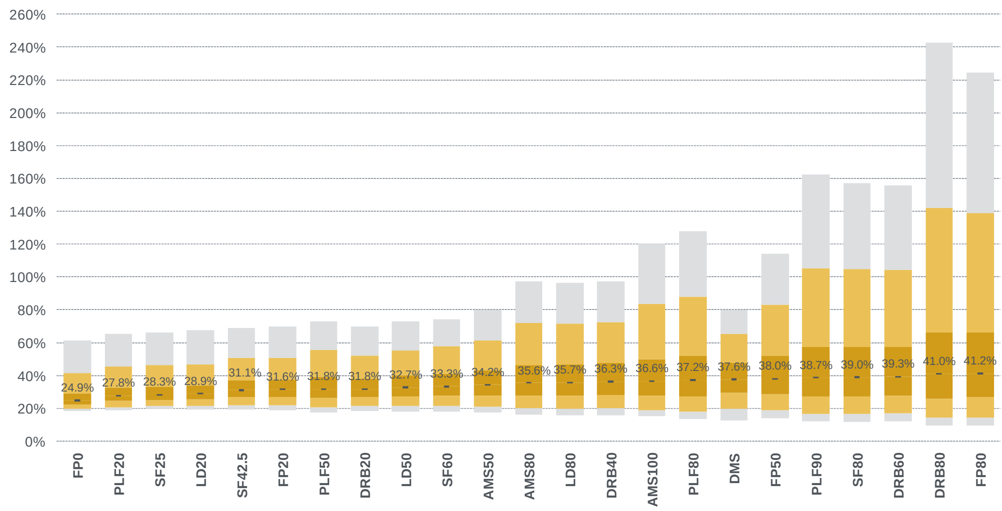
Figure 20: PO 3: risk return diagram (shortfall probability)



Most of the investment strategies have a shortfall probability below 2%. Investment strategies with low equity exposure are inefficient, the other strategies – also the piecewise linear functions – build an efficient frontier. The risk in term of having an amount of benefit payments less than the amount of contributions is very high for the DRB80 and FP80 investment strategies, compared to the others, which is mainly due to the high equity exposure.

Finally, we look at the percentile plot of replacement rates. Basically, we can derive the same conclusions as for the other payout options in the Sections above.

Figure 21: PO 3: percentile plot of replacement rates

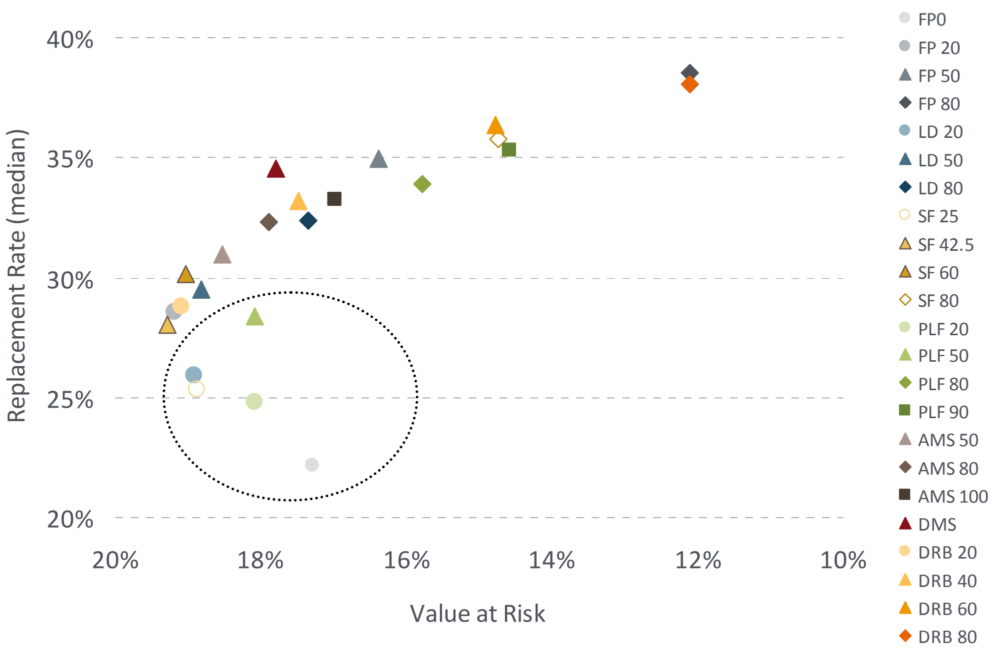


In addition, we can use this figure to compare selected investment strategies. As shown in *Table 3* strategies FP50 and SF80 are very similar with respect to their volume weighted share is risky assets (50.0% resp. 50.4%). However, this does not mean that their risk-return profile is the same. *Figure 21* shows the different upside potential which is much higher for SF80. In contrast, the other figures in this Section give evidence that the associated risk is higher, too. Hence, a volume weighted share in risky assets does not seem to be an appropriate measure to limit risk exposure.

5.5 Withdrawal Program with Variable Payment

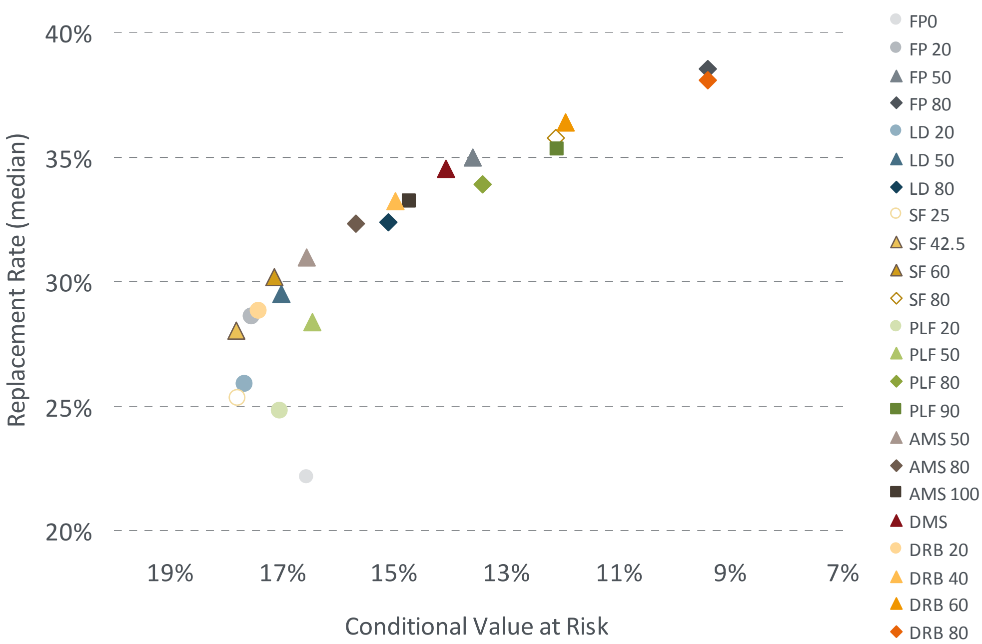
In this Section we, again, investigate the results for a withdrawal program as a payout option, i.e. the investment strategy is relevant for the savings and the payout phase. Contrary to the withdrawal program in Section 5.4 the payout in this Section is life-long and variable as the payments depend on the portfolio value.

Figure 22: PO 4: risk return diagram (VaR)



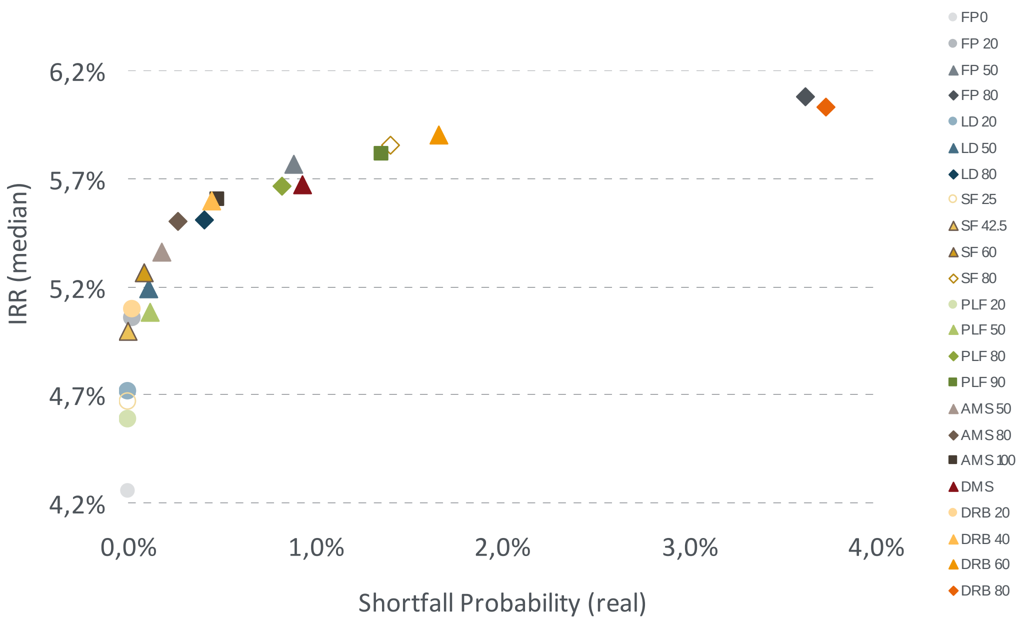
We basically observe the same results as for the withdrawal program with fixed payment in Section 5.4: Most of the investment strategies are located along a curve; investment strategies with low equity exposure and the piecewise linear function investment strategies appear to be inefficient (cf. points in dotted circle). In fact, all strategies with no equity expsoure during the retirement phase are inefficient; DMS has a preferable risk-return profile in this case.

Figure 23: PO 4: risk return diagram (CVaR)



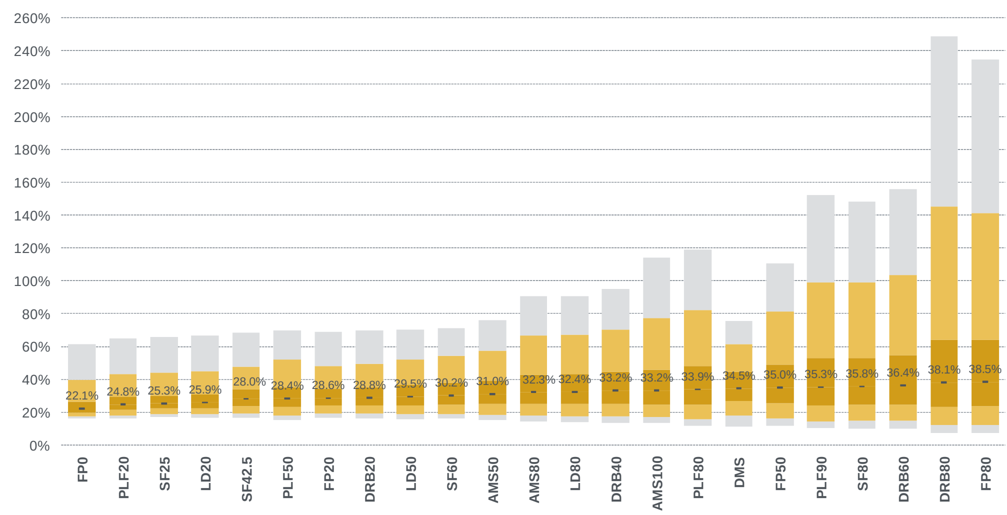
Also regarding the next risk-return diagrams (cf. *Figure 23* and *Figure 24*) results do not differ significantly from the results given in Section 5.4.

Figure 24: PO 4: risk return diagram (shortfall probability)



Finally, we also plot the percentiles of the replacements for each investment strategy.

Figure 25: PO 4: percentile plot of replacement rates

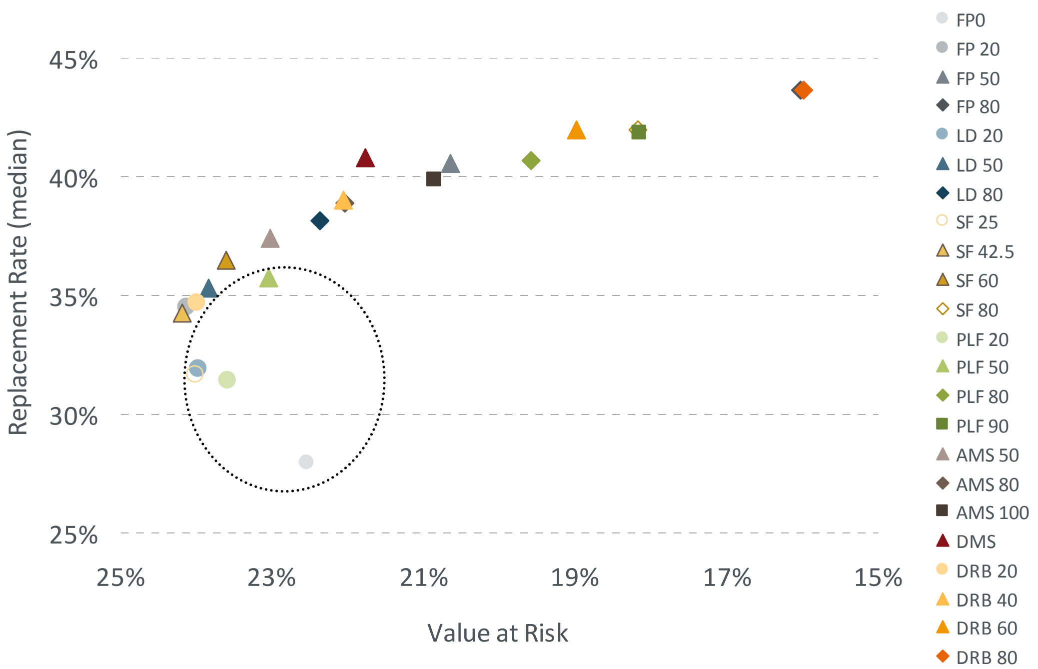


5.6 Withdrawal Program plus Deferred Annuity

The last analyzed payout option includes elements of both payout options 2 and 4 as a withdrawal program with variable payments is combined with an inflation-linked annuity. Hence, the results are similar to the results for payout option 2 and 4.

The following figure shows a risk-return diagram where the risk is the 5% percentile of the replacement rate distribution and the return is the median of the replacement rate distribution.

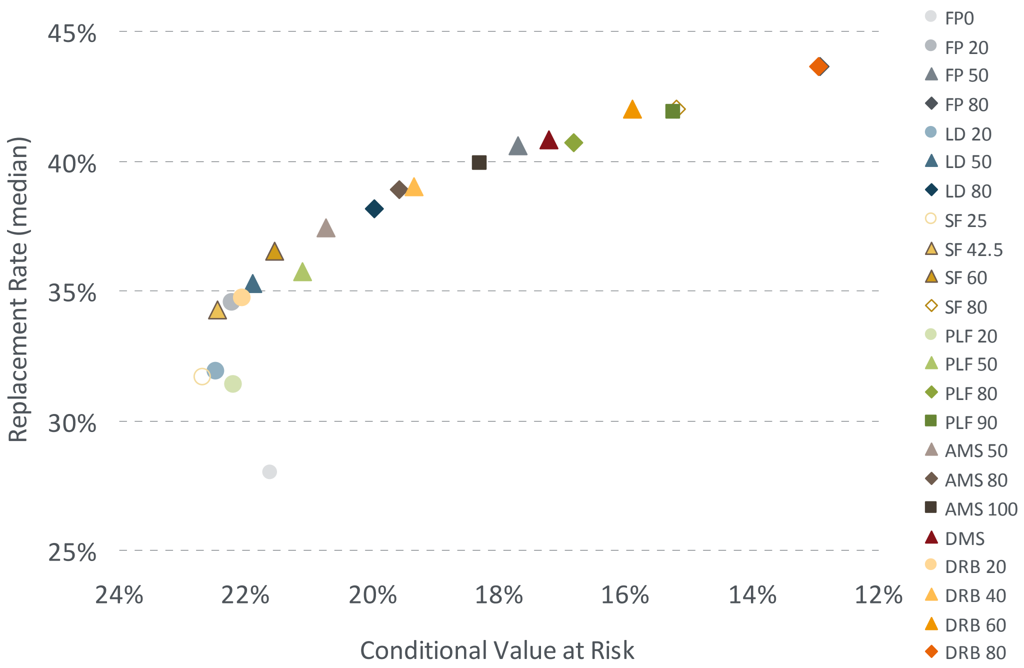
Figure 26: PO 5: risk return diagram (VaR)



The investment strategies are located along a line, which resembles an efficient frontier. Given these risk and return measures investment strategies with low equity exposure appear to be inefficient. This holds also for PLF50. The dynamic life-cycle strategy (DMS) has a preferable risk-return profile providing a higher return for a given risk.

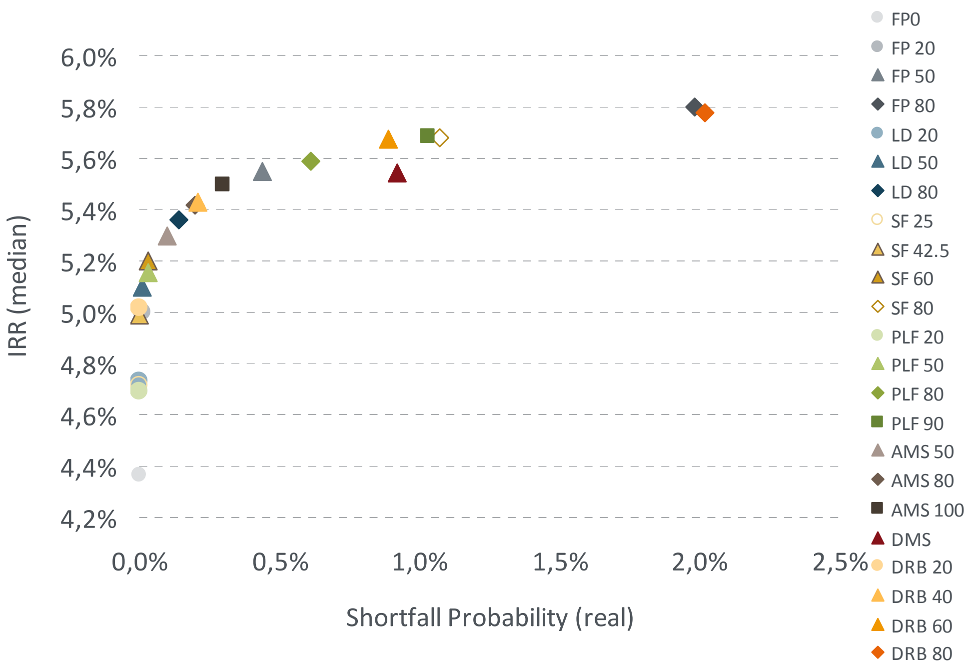
The risk-return diagram in *Figure 27* uses the conditional value at risk. Basically all investment strategies build the efficient frontier. Again, investment strategies with low equity exposure are inefficient.

Figure 27: PO 5: risk return diagram (CVaR)



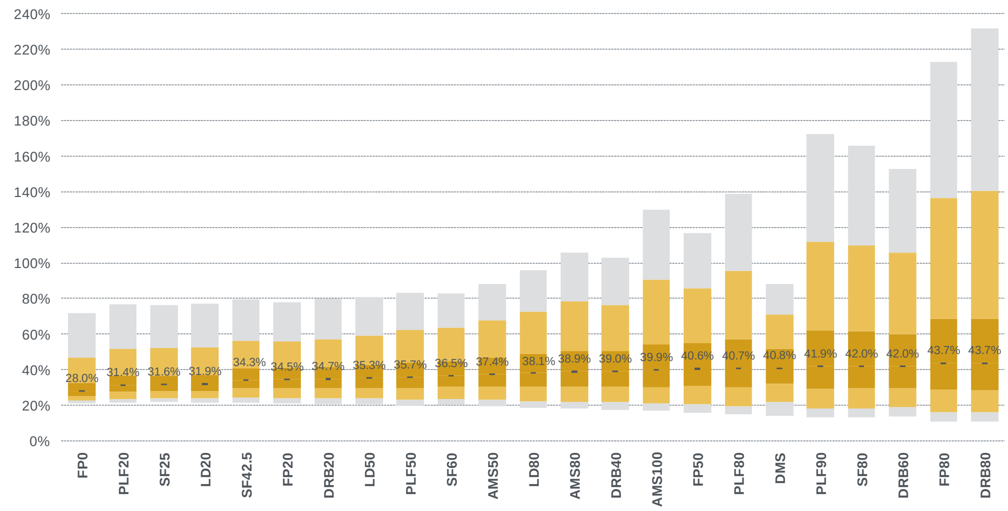
The same statements can be given regarding the next risk-return diagram. In addition, we find DMS located under the efficient frontier as already observed for payout option 2 in *Figure 16*.

Figure 28: PO 4: risk return diagram (shortfall probability)



Finally, more details regarding the distribution of replacement rates are given in the figure below where percentiles for all investment strategies are plotted.

Figure 29: PO 5: percentile plot of replacement rates

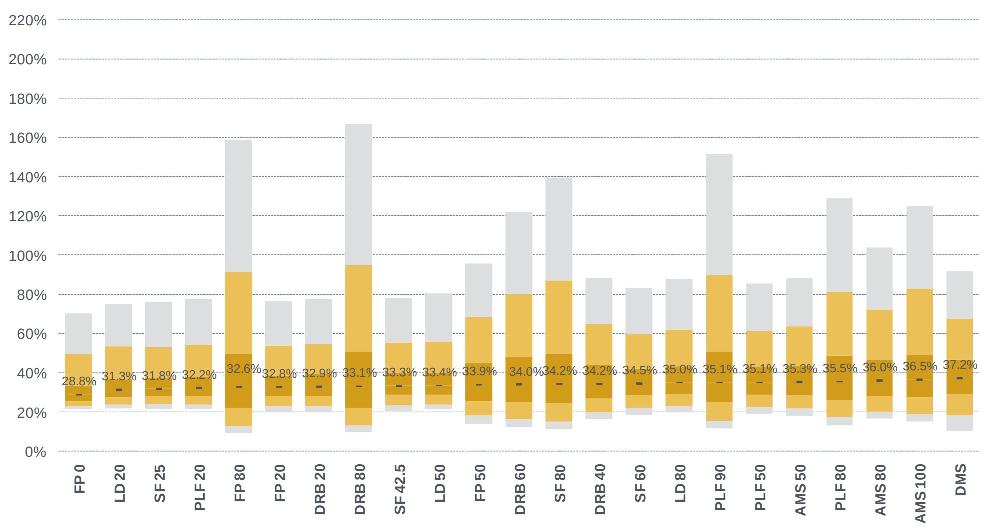


5.7 Conditional Scenarios

A major concern of plan members (and regulators as well) is, that due to an adverse capital market event shortly before retirement the accumulated wealth of the savings plan is significantly reduced so that the employee either can not retire as planned or has to live on a critically lower retirement income. In this subsection we therefore will investigate the impact of an equity shock shortly before retirement to assess what investment strategies can cope with such a situation. First, we identify those of the 10,000 scenarios where at least one drop of equity of 25% or more within any period of 12 months shortly before retirement occurs. "Shortly before retirement" in this analysis means a period of 5 years, i.e. from the age 60 to 65. This leaves us with 2763 scenarios where we observed such a dramatic capital market event. Second, we analyze the impact of the shock for each for the 23 investment strategies by plotting their emperical distribution of replacement rates using these conditional scenarios.

In this analysis we assume PO1 (constant annuity) as payout option as the focus here is to assess the impact of a shock at the end of the savings phase, i.e. having only a short period left to recover. Hence, the payout phase is not considered.

Figure 30: Equity shock of 25%



In the figure above investment strategies are in the order of the median starting from the left with the strategy which has the lowest median. The highest median can be observed for the dynamic multishape strategy DMS, i.e. over all stress scenarios DMS provides on average the highest replacement rate. Next best are two AMS stategies where the glide path of equity exposure is derived from DMS.

Strategies with the highest median in the unconditioned 10,000 scenarios (as FP80, DRB80, SF80 and PLF90) have a significantly lower median. Still theses strategies result in very volatile results.

6 Conclusions and Discussion

In this study we performed a simulation analysis on the risk-return characteristics of DC pension plans and their retirement income generated under 5 different payout options. The goal of the analysis was also to provide a systematic and quantitative basis for assessing 23 commonly found DC plan investment strategies with respect to their qualification as default investment options, designed for people who do not make an active choice of investment.

Building up on a standardized DC pension plan setting, a large variety of investment strategies are investigated and compared. As life-cycle concepts are predominant in target date funds, several strategic and dynamic life-cycle strategies are being modeled. Also fixed portfolio allocation and dynamic risk budget strategies are implemented, aiming to cover different approaches for long term pension and savings investment strategies.

As a standard in the literature, the replacement rate distribution is utilized to represent the main risk and return characteristics of a DC plan. Different views on the relevant measure of risk however underscore the high complexity of choosing an appropriate default investment strategy, depending additionally e.g. on asset universe, contribution rate, payout design.

The simulation results show that there is not one unique or dominating investment solution to be recommended as the default option. Alternative strategies are located along a curve resembling an efficient frontier. Thus the assessment on suitability depends on individual risk bearing preferences or capabilities. Some strategies, e.g. such the ones with a very low exposure to equities, however, are inefficient with respect to the considered risk measures given the long investment horizon. In general, life-cycle investment strategies and dynamic strategies such as the DMS or the DRB (which take path depended decisions and show a more asymmetric retirement income distribution) deliver comparable statistical results.

From the collection of outcomes some further points can be pointed out:

We can observe a wide dispersion of outcomes around the median replacement rate. This indicates that significant disparities in retirement income can occur between plan members -- even for ones of a cohort with similar profiles. In one scenario based on an AMS(80) strategy, retirees could get 100% replacement rate. Those retiring a few years later, after a severe market trubulence (like we saw when the "dot.com market bubble" bursted), could expect significantly less retirement income, in the range of 20% of their last salary only. The controlling of investment risk plays an important factor for the outcome of DC plension plan.

Concerning risk limitations, quantitative investment regulations (e.g. investment ceilings, minimum rate of return, VAR or CVAR / expected shortfall) can be used to restrict investment policies to those that provide a certain combination of potential retirement income and risks. Risk adverse plan members (as well as regulators and supervisors) will aim at policies that reduce the downside risk or that minimise the risk of unfavourable outcomes from DC plans. Such regulations come at the cost of renouncing potentially higher replacement rates that are attainable but at a higher risk of unfavourable retirement income outcomes.

Setting a volume weighted share in equity as a restriction to measure the equity exposure, is more adequate in terms of replacement rate, rather than focusing on a strict investment restriction like a maximal equity exposure or a time weighted average. In general, investment strategies with an (VWS) equity exposure lower than 8% are inefficient in term of replacement rate, since they do not allow to benefit enough from the upside potential of the markets. The outcomes also underline that life-cycle investment strategies with no equity exposure during the payment phase are inefficient in term of replacement rate, if the payout option still offers the possibility to invest in equity markets.

Most of analyzed investment strategies have a shortfall probability below 2% (especially for payout options designed to be dependent on investment returns during the payout phase), which means that portfolio returns are high enough to provide the retiree with benefit payments. Nevertheless for aggressive investment strategies, this shortfall probability is quite high, in particular for full withdrawal program payout options (e.g. withdrawal program with fixed payments), where this probability can achieve 5.6%.

At first sight, DMS is a clear outlier in the range of the deterministic strategies and seems to be satisfying. Having a closer look shows the benefits such a strategy can contribute. For example, DMS has the highest 25% percentile of all strategies. This advantage comes at a cost of having a significantly lower 75% percentile compared to its neighbors. Depending on the preference of an individual member or even the regulator this could be favorable. Considering the risk of the DMS, all strategies with a higher median have a lower 5% percentile. Yet, this advantage seems not to hold for the 1% percentile.

The design of the default option should depend on various factors, including the length of the contribution and accumulation period and the type of benefit pay-out allowed or regulated as the default option. From the analysis we find that most life cycle strategies could be considered as appropriate default investment strategies as they provide protection for those close to retirement in the case of a negative shock to the stock market. Nevertheless, a high volatility of retirement income due to investment risk remains.

The conditional scenario analysis shows that a dynamic investment strategy such as DMS can protect the retiree better in case of such large equity shocks short before retirement than life-cycle strategies can do. The median replacement rate of all scenarios with an equity drop of 25% within a 12 month period in the last 5 years before retirement is the highest for all analyzed investment strategies.

Extensions of the analysis are suggested to do with respect to relaxing the underlying model assumptions. One aspect would be to include the impact of different wage profiles and intermittent contributions and differentiated between blue collar and white collar worker profiles. Different contribution-age profiles and their interactions with investment strategies could be investigated. More general, various forms of human capital risk could be addressed and additional practical aspects considered such as types and levels of costs and fees going along with the different investment alternaitves.

Finally, it would be valuable to introduce better ways of communicating the various dimensions of DC plan risk to the schemes' members.

Beyond the scope of this study but topics for further research are other ways to address and limit pension investment risk for default option plans: The introduction of explicit guarantees on minimum (real or nominal) replacement rates are critical elements in the DC pension regulations of some contries. Buying forward starting annuities through one's working life could also help to reduce retirement income uncertainties due to investment results. However, before stipulating any guarantees or new investment option one should be clear about the consequences of protection of the plan members via guarantees as this will also reduce the upside potential. Consequently, the cost of guarantees should be analysed as a first step.

7 Appendix

7.1 Economic Scenario Generator

Scenario analyses have become a powerful tool to assess complex financial decision situations. With increased computational power, intensive analyses with many economic variables over several time steps have meanwhile become a feasible task.

The quality of the results and conclusions drawn upon depend heavily on the economic scenarios assumed within the analysis. So the underlying concept of the scenario generation is crucial. Different Economic Scenario Generators (ESGs) have been developed and described in the literature.[24] Among the available ESGs, three classes can be distinguished in terms of the underlying model:

- The econometrics-based models.
- The pricing-based models.
- The hybrid models.

The first one is based on statistical and econometric theory.[25] The main advantage of this ESG class is its simplicity. However, a fully historical-based method might not be suitable or at least not robust enough for forecasting purposes because it depends on the number of available observations. Besides, this kind of models cannot be used to price financial instruments since simulations under the risk-neutral measure are not possible.

The second class brings a solution for the latter issues. Indeed, these models are based on stochastic mathematical tools which allow risk-neutral calculations. The pricing-based models[26], unlike the econometrics-based ones, can accommodate for both expressing historical information as well as future expectation. However, risk factors, e.g. inflation or interest rate, may not be in line with global understanding of the market since the focus of these models is put on pricing.

[24] A.D. Wilkie, M.A., F.F.A., F.I.A. (1995), 'More on a stochastic asset model for actuarial use', British Actuarial Journal. 1, 777-964.

J. Hibbert, P. Mowbray, C. Turnbull (2001), 'A stochastic asset model & calibration for long-term financial planning purposes'.

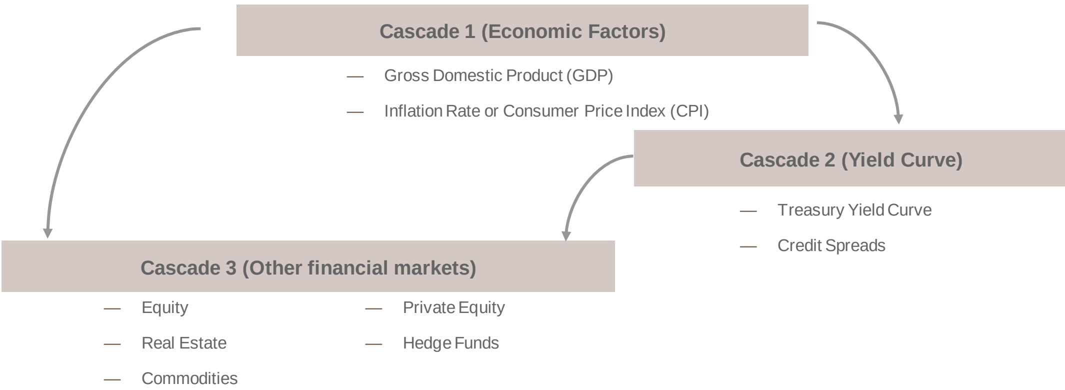
R. Zagst, S. Antes, B. Schmid, M. Ilg (2008), 'Empirical evaluation of hybrid defaultable bond pricing models', Journal of Applied Mathematical Finance.

[25] The Wilkie Investment Model, very popular in the United Kingdom, is a prominent representative of these kinds of models. The first version was released in 1986 and the second in 1995.

[26] The Barrie and Hibbert Model is a prominent representative of these kinds of models. Insights have been published in 2001.

Thus a third class of ESGs evolved. The hybrid model, where risklab ESG[27] is one example, tries to combine the main advantages of both previous classes. Indeed the model, which is implemented in the risklab ESG, aims to overcome the drawback of the pricing-based ESGs by including the influence of macroeconomic variables in the integrated modelling framework. It combines both statistical and financial theory in the sense that it uses observable financial variables like inflation to describe the evolution of other economic or pricing variables that are not necessarily observable. To achieve this kind of sophistication, the underlying processes are modelled by SDEs with parameters that are estimated gradually following a cascade structure as shown in *Figure 31*.

Figure 31: Cascade structure used to model the risklab ESG



The advantage of such integrated models is to allow the estimation of a complex market model with different interdependent factors step by step. Thus if inflation is a top generating economic factor, then all the factors modelled below it might depend on inflation. All the processes in Cascade 1 and Cascade 2 are modeled with a mean reversion property in mind which means that the processes tend towards their long-term mean. Such behaviour can e.g. be observed in the market for interest rates or the GDP.

Cascade 1 deals with the macroeconomic parameters such as the Gross Domestic Product (GDP) and the inflation. The dynamics of the GDP growth rate r_ω and the dynamics of the inflation rate r_i are respectively given by the following Vasicek model[28]:

$$dr_\omega(t) = (\theta_\omega - a_\omega r_\omega(t))dt + \sigma_\omega dW_\omega(t) \tag{1}$$

$$dr_i(t) = (\theta_i - a_i r_i(t))dt + \sigma_i dW_i(t) \tag{2}$$

[27] The risklab economic scenario generator is a proprietary ESG which is enhanced in cooperation with Prof. Dr. Rudi Zagst, Professor of Mathematical Finance at the Munich University of Technology.

[28] The Vasicek model describes the evolution of rates and was the first one to capture the mean reversion characteristic. It was introduced in 1977 by Oldrich Vasicek.

where $W_\omega = (W_\omega(t))_{t\geq 0}$ and $W_i = (W_i(t))_{t\geq 0}$ are Wiener processes. The mean reversion levels are given by $\frac{\theta_\omega}{a_\omega}$ for (1) and $\frac{\theta_i}{a_i}$ for (2).

Cascade 2 deals with the treasury yield curves and the credit spreads. Concerning the treasury yield curve the real short rate r_R dynamics is given by a two-factor Hull-White model[29].

$$dr_R(t) = (\theta_R(t) + b_{R\omega}\omega(t) - a_R r_R(t))dt + \sigma_R dW_R(t) \tag{3}$$

where $W_R = (W_R(t))_{t\geq 0}$ is a Wiener process. The mean reversion level is given by $\frac{\theta_R(t) + b_{R\omega}\omega(t)}{a_R}$.

Since the nominal short rate is defined as the sum of the real short rate and the inflation short rate, i.e. $r = r_R + r_i$, the dynamics of the nominal short rate can be deduced from (2) and (3).

Concerning the short rate credit spread, its dynamics is given by a three-factor Hull-White model, where one driving factor is the so-called uncertainty index u.

$$du(t) = (\theta_u - a_u u(t))dt + \sigma_u dW_u(t) \tag{4}$$

$$ds(t) = (\theta_s + b_{su}u(t) - b_{s\omega}\omega(t) - a_s s(t))dt + \sigma_s dW_s(t) \tag{5}$$

where $W_u = (W_u(t))_{t\geq 0}$ and $W_s = (W_s(t))_{t\geq 0}$ are Wiener processes. The mean reversion level is given by $\frac{\theta_s + b_{su}u(t) - b_{s\omega}\omega(t)}{a_s}$.

Cascade 3 deals with the equity and the alternatives indexes. The dynamics of the stock return r_E is given by the following stochastic differential equation:

$$dr_E(t) = (\alpha_E + b_{E\omega}\omega(t) - b_{Ei}i(t) + b_{ER}r_R(t))dt + \sigma_E dW_E(t) \tag{6}$$

where $W_E = (W_E(t))_{t\geq 0}$ is a Wiener process.

The dividend yield r_D dynamics is given by a Vasicek model, which means again that the property of mean reversion is being kept:

[29] The Hull-White model was introduced in 1990 by John Hull and Alan White. The Vasicek model is a derived form of the Hull-White model.

$$dr_D(t) = (\theta_D - a_D r_D(t))dt + \sigma_D dW_D(t) \tag{7}$$

where $W_D = (W_D(t))_{t \geq 0}$ is a Wiener process. The mean reversion level is given by $\dfrac{\theta_D}{a_D}$.

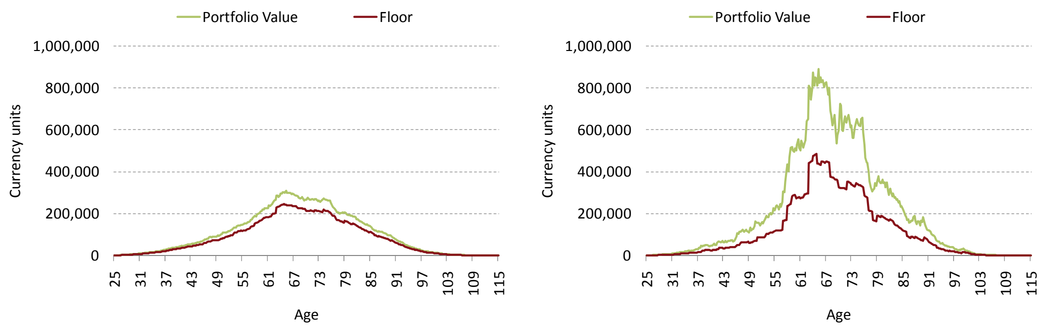
7.2 Dynamic Asset Allocations

Both dynamic investment strategies are developed by risklab. The dynamic risk budget is a strategy developed only for the need of the study.

7.2.1 Dynamic Risk Budget

*Figure 32*illustrates the dynamics of the portfolio value and the floor (during the contribution and the payout phases) given one scenario of the payout option 4 for the DRB20 and DRB80 investment strategies.

Figure 32: Floor and asset dynamics



In the following we intend to clarify this strategy further by giving an example of a simulation path for the DRB 40 investment strategy (payout option 5) representing the asset allocation dynamics. *Figure 33* and *Figure 34*illustrate the dynamic allocation process implied by the investment strategy for a given age interval. *Figure 33* illustrates the basic principle of the strategy, which is defined by a more aggressive allocation (which means more allocation in equity) when the risk budget is getting high and vice versa. The y-axis on the right side of the graph shows the value of the current risk budget, which is re-calculated in each time step of the simulation, whereas the y-axis on the left side of the graph represents the asset allocation.[30]

[30] In this scenario the allocation in cash and inflation-linked bonds is zero and almost zero, respectively.

Figure 33: Asset allocation dynamics

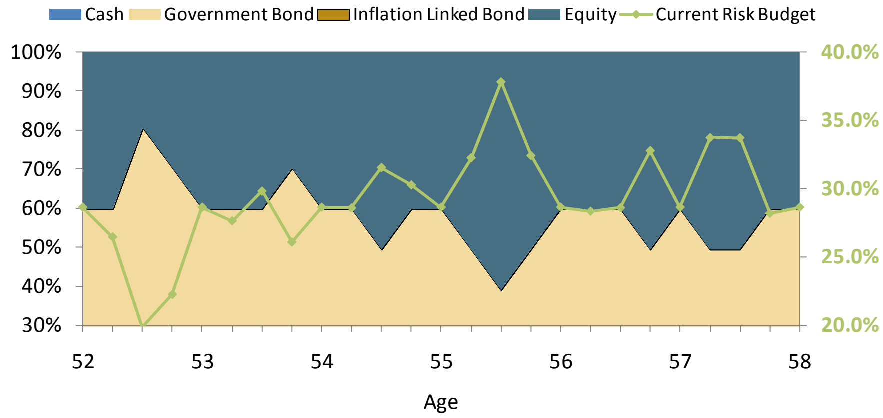
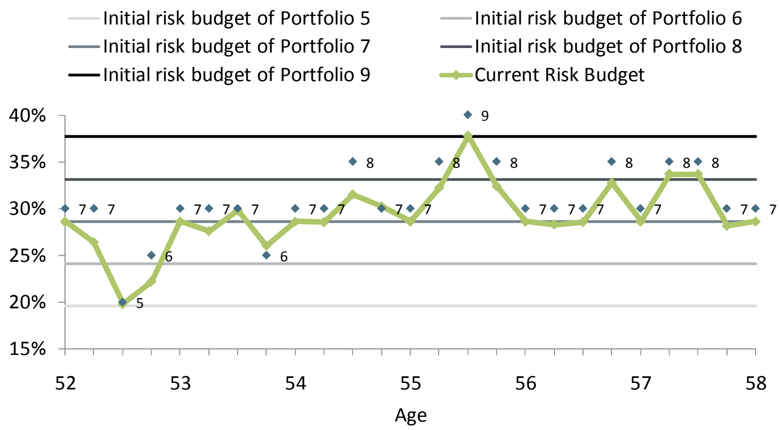


Figure 34 illustrates the portfolio selection process, which is defined by the comparison of the current risk budget with the 5 initial risk budgets attributed to the 5 valid portfolios. The portfolio whose initial risk budget comes closest to the current risk budget is selected as the current/new portfolio allocation.

Figure 34: Portfolio selection dynamics



7.2.2 Dynamic Life-cycle

As shown in *Figure 35*, various factors are modeled within the framework and have an impact on the optimization. The human capital is accounted for in terms of a stochastic income process which is subject to the level of education of the individual, but also to the market environment.[31] Furthermore, the lifetime itself of the person is stochastic. This feature captures the so-called longevity risk, i.e. the risk to outlive one's assets. Savings within this framework are assumed a deterministic portion of the income. As the income is stochastic, the saving will also be – despite the deterministic savings rate. The wealth is defined as current value of the portfolio.

[31] However, within the framework applied here, the stochastic is only driven by the inflation.

Furthermore, the markets themselves can have an impact on the optimal allocation. As the performance of the underlying assets in the portfolio is crucial to the question whether to meet the goal to reach to the pre-defined requirements, the strategy can react and adjust according to the past performance of the portfolio.

Resulting from the optimization is an allocation rule which, for every age, gives the optimal allocation depending on the income and the financial wealth of the person[32] as shown in *Figure 36*. Thus, the investment strategy accounts for movements in the markets and provides the optimal allocation in every market environment. Moreover it is tailored to the financial condition of every single individual. When adjusting the allocation with respect to individual investor characteristics is not possible, we can also, we can also derive an average allocation over time. This is what we use for the AMS investment strategies.

Figure 35: Step 1: optimisation

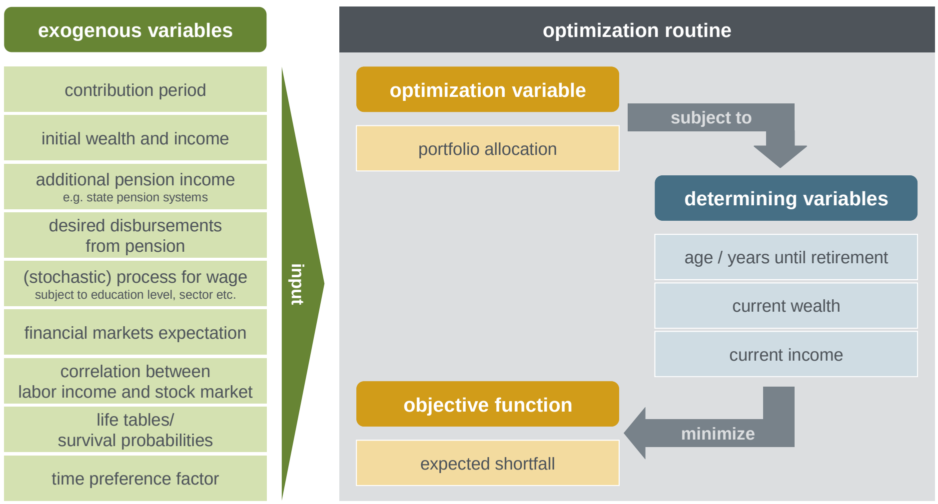
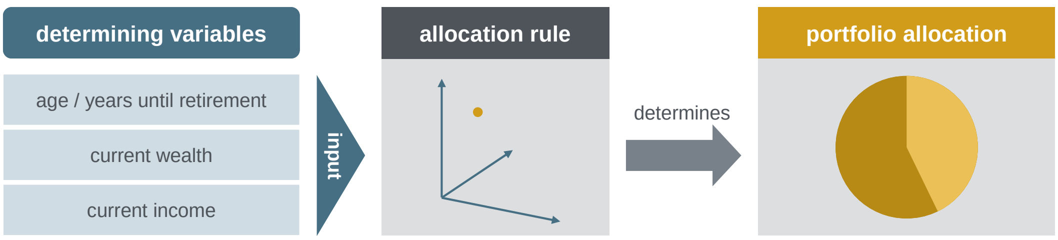


Figure 36: Step 2: Application



[32] This is again subject to the performance of the underlying assets.

7.3 Summary Table

Table 5represents the replacement rate distributions given their quantile values, for the 5 payout options and the 23 investment strategies.

- PO1: Constant life annuity beginning at age 65.
- PO2: Inflation-linked life annuity beginning at age 65.
- PO3: Withdrawal program with fixed payments beginning at age 65 and ending at 89.
- PO4: Withdrawal program with variable payments beginning at age 65.
- PO5: Withdrawal program with variable payments from age 65 up to age 79 + deferred variable life annuity beginning at age 80.

Table 5 Replacement Rate Distributions

Payout Option: Constant Life Annuity

RepRate	FP0	FP20	FP50	FP80	LD20	LD50	LD80	SF25	SF42.5	SF60	SF80	PLF20	PLF50	PLF80	PLF90	AMS50	AMS80	AMS100	DRB20	DRB40	DRB60	DRB80	DMS
CVaR-1%	20.9%	19.8%	14.4%	9.7%	21.1%	20.5%	19.2%	21.2%	20.1%	18.8%	11.8%	21.0%	18.7%	13.5%	11.9%	17.8%	16.5%	15.2%	19.8%	16.3%	12.7%	9.9%	10.8%
Q-1%	21.6%	21.2%	16.2%	11.5%	21.9%	21.7%	20.6%	22.2%	21.6%	20.5%	13.6%	21.9%	20.5%	15.2%	13.6%	19.6%	18.2%	16.9%	21.1%	18.0%	14.4%	11.6%	14.0%
CVaR-5%	22.1%	22.4%	17.8%	13.1%	22.8%	22.6%	21.8%	23.0%	22.6%	21.7%	15.2%	22.8%	21.6%	16.9%	15.3%	20.8%	19.6%	18.3%	22.2%	19.5%	16.1%	13.2%	17.1%
Q-5%	23.1%	24.2%	20.4%	16.0%	24.0%	24.2%	23.7%	24.4%	24.4%	23.7%	18.0%	24.1%	23.5%	19.6%	18.0%	23.0%	22.0%	20.8%	24.1%	21.9%	18.7%	16.0%	21.7%
Q-25%	25.9%	29.6%	30.1%	27.6%	27.7%	29.2%	29.9%	28.4%	29.7%	30.4%	28.9%	28.3%	30.2%	29.7%	29.0%	30.4%	30.4%	30.1%	29.5%	30.0%	29.0%	27.3%	32.0%
Median	28.8%	34.6%	39.6%	41.8%	31.4%	33.8%	35.8%	32.2%	34.6%	36.5%	41.3%	32.3%	36.6%	40.7%	41.3%	37.5%	38.8%	39.7%	34.7%	38.1%	40.4%	41.7%	40.4%
Mean	31.7%	36.9%	43.8%	52.0%	34.1%	36.4%	38.7%	34.8%	37.1%	39.3%	49.2%	34.9%	39.4%	46.8%	49.7%	40.5%	43.0%	45.5%	37.2%	41.7%	47.0%	52.5%	42.8%
Q-75%	33.6%	41.1%	52.7%	64.0%	36.6%	40.1%	44.0%	37.7%	41.2%	44.8%	59.9%	37.9%	45.0%	56.5%	60.2%	47.0%	50.5%	53.8%	41.5%	49.2%	57.3%	64.3%	50.9%
Q-95%	49.2%	57.0%	82.0%	121.4%	52.7%	56.7%	63.4%	53.7%	57.6%	64.1%	106.0%	54.0%	64.6%	94.6%	109.7%	68.3%	78.3%	90.3%	58.2%	73.4%	97.1%	124.0%	70.8%
Q-99%	75.5%	80.8%	110.3%	191.5%	79.6%	82.6%	85.7%	80.2%	82.2%	85.8%	161.2%	80.0%	86.2%	137.9%	170.1%	90.3%	108.5%	130.2%	81.9%	98.8%	144.6%	204.3%	89.8%
Volatility	11.6%	12.2%	20.0%	37.6%	11.9%	12.2%	13.7%	11.8%	12.2%	13.7%	30.7%	12.0%	14.0%	25.5%	31.9%	15.0%	18.7%	23.6%	12.5%	17.1%	26.6%	40.0%	16.0%
Skewness	7.1	3.9	1.7	2.7	5.9	4.1	2.7	5.3	3.5	2.4	2.4	5.3	2.5	2.2	2.5	2.1	2.0	2.4	3.7	1.8	2.1	3.1	1.5
Kurtosis	111.3	44.2	8.6	16.4	83.6	45.0	20.7	65.6	33.2	16.9	13.1	68.3	18.2	11.2	14.6	13.6	10.9	13.5	38.7	9.8	11.3	20.6	10.0
VWS Equity	0.0%	20.0%	50.0%	80.0%	5.3%	13.1%	20.8%	8.3%	17.4%	26.5%	65.5%	8.2%	26.1%	55.7%	65.5%	32.3%	39.3%	45.9%	21.6%	42.1%	62.3%	81.5%	38.1%
TWS Equity	0.0%	20.0%	50.0%	80.0%	10.0%	25.0%	40.0%	14.3%	26.4%	38.5%	72.7%	13.7%	37.3%	67.3%	77.3%	41.0%	56.9%	69.3%	20.9%	40.5%	59.7%	78.0%	61.3%

Payout Option: Inflation Linked Life Annuity

RepRate	FP0	FP20	FP50	FP80	LD20	LD50	LD80	SF25	SF42.5	SF60	SF80	PLF20	PLF50	PLF80	PLF90	AMS50	AMS80	AMS100	DRB20	DRB40	DRB60	DRB80	DMS
CVaR-1%	20.1%	19.4%	14.1%	9.5%	20.4%	19.8%	18.7%	20.5%	19.6%	18.3%	11.5%	20.3%	18.2%	13.2%	11.6%	17.4%	16.0%	14.7%	19.3%	16.0%	12.4%	9.7%	10.5%
Q-1%	20.8%	20.6%	15.8%	11.1%	21.3%	20.9%	20.0%	21.4%	20.9%	19.8%	13.1%	21.3%	19.8%	15.0%	13.2%	18.9%	17.7%	16.4%	20.4%	17.7%	14.0%	11.3%	13.3%
CVaR-5%	21.4%	21.7%	17.4%	12.8%	22.0%	22.0%	21.2%	22.3%	22.0%	21.1%	14.9%	22.1%	20.9%	16.5%	14.9%	20.3%	19.1%	17.9%	21.6%	19.0%	15.7%	12.9%	16.7%
Q-5%	22.4%	23.6%	20.0%	15.5%	23.3%	23.6%	23.1%	23.6%	23.8%	23.1%	17.6%	23.5%	23.0%	19.2%	17.6%	22.6%	21.5%	20.3%	23.5%	21.3%	18.4%	15.6%	21.1%
Q-25%	25.4%	29.0%	29.4%	27.3%	27.3%	28.6%	29.3%	27.9%	29.1%	29.7%	28.4%	27.8%	29.6%	29.2%	28.4%	29.8%	29.8%	29.5%	29.0%	29.4%	28.4%	26.9%	31.4%
Median	28.4%	33.9%	38.9%	41.2%	30.9%	33.2%	35.2%	31.7%	33.9%	35.9%	40.5%	31.8%	36.0%	39.9%	40.5%	36.8%	38.1%	39.0%	34.1%	37.5%	39.8%	41.0%	39.7%
Mean	31.1%	36.2%	43.0%	51.0%	33.5%	35.7%	38.0%	34.2%	36.4%	38.5%	48.3%	34.3%	38.7%	46.0%	48.9%	39.8%	42.2%	44.7%	36.5%	41.0%	46.2%	51.6%	42.0%
Q-75%	33.1%	40.4%	51.8%	62.8%	36.0%	39.6%	43.2%	37.0%	40.5%	44.1%	58.8%	37.2%	44.4%	55.8%	59.3%	46.1%	49.8%	53.4%	40.9%	48.4%	56.2%	63.0%	50.3%
Q-95%	48.1%	55.7%	80.2%	119.4%	51.8%	55.6%	62.2%	52.7%	56.7%	63.0%	104.6%	53.0%	63.6%	93.7%	107.8%	66.7%	77.3%	88.6%	56.7%	72.5%	95.3%	122.2%	69.5%
Q-99%	72.5%	78.9%	111.0%	187.5%	76.4%	80.0%	82.8%	76.2%	80.4%	83.8%	157.0%	77.4%	84.3%	135.5%	164.6%	87.8%	106.4%	128.0%	79.5%	96.4%	140.4%	201.4%	87.6%
Volatility	11.2%	11.9%	19.7%	36.9%	11.6%	11.9%	13.4%	11.5%	12.0%	13.5%	30.1%	11.6%	13.7%	25.0%	31.4%	14.8%	18.4%	23.2%	12.2%	16.8%	26.2%	39.2%	15.7%
Skewness	7.0	3.7	1.7	2.7	5.8	4.0	2.6	5.1	3.4	2.3	2.4	5.1	2.4	2.1	2.5	2.0	2.0	2.3	3.5	1.8	2.1	3.0	1.4
Kurtosis	108.0	41.0	8.4	16.2	80.1	42.0	19.0	62.7	30.9	15.7	12.9	64.7	16.7	11.0	14.4	12.6	10.4	12.9	35.9	9.3	11.1	20.2	9.2
VWS Equity	0.0%	20.0%	50.0%	80.0%	5.3%	13.1%	20.8%	8.3%	17.4%	26.5%	65.5%	8.2%	26.1%	55.7%	65.5%	32.3%	39.3%	45.9%	21.6%	42.1%	62.3%	81.5%	38.1%
TWS Equity	0.0%	20.0%	50.0%	80.0%	10.0%	25.0%	40.0%	14.3%	26.4%	38.5%	72.7%	13.7%	37.3%	67.3%	77.3%	41.0%	56.9%	69.3%	20.9%	40.5%	59.7%	78.0%	61.3%

Payout Option: Withdrawal Program with Fixed Payment

RepRate	FP0	FP20	FP50	FP80	LD20	LD50	LD80	SF25	SF42.5	SF60	SF80	PLF20	PLF50	PLF80	PLF90	AMS50	AMS80	AMS100	DRB20	DRB40	DRB60	DRB80	DMS
CVaR-1%	18.1%	17.6%	12.3%	7.9%	18.3%	16.6%	14.1%	18.7%	17.9%	16.7%	10.4%	18.0%	16.2%	12.0%	10.5%	15.8%	14.7%	13.5%	17.4%	14.0%	10.6%	8.0%	9.7%
Q-1%	18.6%	18.9%	14.0%	9.4%	19.2%	18.1%	15.9%	19.5%	19.2%	18.1%	11.9%	18.8%	17.6%	13.6%	12.1%	17.5%	16.4%	15.3%	18.6%	15.7%	12.2%	9.4%	12.7%
CVaR-5%	19.1%	20.0%	15.9%	11.4%	20.1%	19.3%	17.2%	20.2%	20.2%	19.5%	13.8%	19.6%	18.7%	15.2%	13.8%	18.8%	17.7%	16.6%	19.9%	17.4%	14.2%	11.4%	15.6%
Q-5%	19.9%	21.8%	18.7%	14.3%	21.4%	21.3%	19.6%	21.4%	21.8%	21.4%	16.7%	20.8%	20.5%	17.8%	16.5%	20.9%	20.0%	19.0%	21.7%	20.0%	17.1%	14.3%	19.9%
Q-25%	22.4%	27.0%	28.4%	26.6%	25.4%	27.3%	27.8%	25.0%	26.7%	27.7%	27.3%	24.5%	26.3%	27.4%	27.1%	27.7%	27.8%	27.6%	26.9%	28.1%	27.5%	26.1%	29.5%
Median	24.9%	31.6%	38.0%	41.2%	28.9%	32.7%	35.7%	28.3%	31.1%	33.3%	39.0%	27.8%	31.8%	37.2%	38.7%	34.2%	35.6%	36.6%	31.8%	36.3%	39.3%	41.0%	37.6%
Mean	27.1%	33.5%	42.8%	54.4%	31.0%	34.9%	39.4%	30.4%	33.1%	35.7%	47.0%	29.9%	34.1%	43.1%	47.0%	36.8%	39.4%	41.9%	33.8%	39.9%	47.2%	55.3%	39.5%
Q-75%	28.8%	37.6%	51.9%	66.3%	33.9%	40.0%	46.8%	33.0%	37.0%	41.0%	57.3%	32.5%	39.2%	52.1%	57.2%	43.1%	46.4%	49.7%	38.0%	47.6%	57.2%	66.4%	47.7%
Q-95%	41.4%	50.9%	83.2%	139.2%	46.9%	55.4%	71.4%	46.4%	50.7%	58.0%	104.8%	45.6%	55.8%	88.2%	105.2%	61.5%	72.0%	83.4%	52.0%	72.4%	104.3%	142.1%	65.5%
Q-99%	61.4%	69.7%	114.1%	225.8%	67.7%	72.9%	96.3%	66.2%	68.9%	74.2%	157.1%	65.8%	72.8%	127.7%	162.7%	80.1%	97.4%	120.4%	70.1%	97.5%	156.1%	243.7%	79.9%
Volatility	9.2%	10.2%	21.1%	48.3%	9.7%	11.4%	17.0%	9.5%	10.1%	11.9%	30.5%	9.6%	11.7%	23.9%	31.4%	13.2%	17.0%	21.9%	10.5%	17.0%	30.8%	54.0%	14.4%
Skewness	6.4	2.6	1.9	6.0	4.2	1.9	1.7	4.4	2.7	1.8	2.6	4.6	2.1	2.2	2.8	1.7	1.9	2.4	2.4	1.7	3.7	9.7	1.0
Kurtosis	88.5	19.9	10.6	119.2	44.4	11.9	8.7	46.8	19.6	9.8	16.2	51.7	13.5	12.2	18.5	8.7	9.4	14.0	18.0	9.1	47.5	294.6	6.1
VWS Equity	0.0%	20.0%	50.0%	80.0%	7.0%	17.7%	28.2%	6.4%	14.6%	22.7%	50.4%	3.5%	12.1%	36.8%	46.5%	25.5%	29.8%	33.4%	21.5%	42.0%	62.3%	81.5%	31.9%
TWS Equity	0.0%	20.0%	50.0%	80.0%	10.0%	25.0%	40.0%	10.8%	21.3%	31.7%	60.6%	8.6%	23.8%	50.5%	60.5%	33.4%	44.3%	52.6%	21.5%	41.6%	61.4%	80.3%	50.1%

Table cont.

Payout Option: Withdrawal Program with Variable Payment

RepRate	FP0	FP20	FP50	FP80	LD20	LD50	LD80	SF25	SF42.5	SF60	SF80	PLF20	PLF50	PLF80	PLF90	AMS50	AMS80	AMS100	DRB20	DRB40	DRB60	DRB80	DMS
CVaR-1%	15.6%	15.3%	10.3%	6.2%	16.1%	14.5%	12.2%	16.4%	15.8%	14.6%	8.9%	15.7%	14.2%	10.4%	9.0%	13.9%	12.9%	11.8%	15.1%	11.8%	8.7%	6.3%	8.7%
Q-1%	16.1%	16.5%	11.8%	7.5%	16.8%	15.9%	13.8%	17.1%	16.8%	16.0%	10.2%	16.4%	15.5%	11.9%	10.4%	15.4%	14.4%	13.4%	16.3%	13.4%	10.0%	7.5%	11.4%
CVaR-5%	16.6%	17.6%	13.6%	9.4%	17.7%	17.0%	15.1%	17.8%	17.8%	17.1%	12.1%	17.0%	16.4%	13.4%	12.1%	16.6%	15.7%	14.7%	17.4%	15.0%	11.9%	9.4%	14.1%
Q-5%	17.3%	19.2%	16.4%	12.1%	18.9%	18.8%	17.4%	18.9%	19.3%	19.0%	14.7%	18.1%	18.1%	15.8%	14.6%	18.5%	17.9%	17.0%	19.1%	17.5%	14.8%	12.1%	17.8%
Q-25%	19.6%	24.2%	25.4%	23.6%	22.6%	24.3%	24.9%	22.2%	23.9%	24.8%	24.7%	21.5%	23.4%	24.6%	24.3%	24.9%	24.9%	24.8%	24.1%	25.2%	24.6%	23.2%	26.6%
Median	22.1%	28.6%	35.0%	38.5%	25.9%	29.5%	32.4%	25.3%	28.0%	30.2%	35.8%	24.8%	28.4%	33.9%	35.3%	31.0%	32.3%	33.2%	28.8%	33.2%	36.4%	38.1%	34.5%
Mean	24.6%	30.6%	40.0%	52.4%	28.2%	31.8%	36.0%	27.7%	30.2%	32.6%	43.5%	27.1%	30.9%	39.5%	43.3%	33.6%	36.0%	38.4%	30.9%	37.0%	44.7%	53.4%	36.5%
Q-75%	26.2%	34.5%	48.8%	64.1%	30.8%	36.6%	43.1%	30.1%	33.8%	37.7%	53.1%	29.4%	35.4%	48.0%	52.9%	39.4%	42.7%	45.7%	35.0%	44.4%	54.7%	64.0%	44.3%
Q-95%	39.6%	48.2%	81.5%	141.4%	44.9%	51.9%	67.0%	44.1%	47.8%	54.3%	99.3%	43.3%	51.9%	82.4%	99.1%	57.3%	66.9%	77.2%	49.3%	70.2%	103.4%	145.1%	61.6%
Q-99%	61.5%	68.8%	110.5%	234.8%	66.7%	70.5%	90.9%	65.8%	68.3%	71.1%	148.4%	65.0%	69.9%	118.8%	152.4%	76.0%	90.7%	114.0%	70.0%	95.3%	156.0%	249.5%	75.6%
Volatility	9.9%	10.3%	21.3%	49.3%	10.1%	11.3%	16.3%	10.0%	10.3%	11.7%	29.4%	10.2%	11.6%	22.5%	30.0%	12.8%	16.2%	20.7%	10.7%	17.1%	31.4%	54.4%	14.0%
Skewness	7.1	2.9	2.0	5.0	4.8	2.3	1.8	5.1	3.1	2.0	2.7	5.3	2.6	2.3	3.0	1.8	1.9	2.4	2.8	1.8	3.4	7.0	1.2
Kurtosis	106.7	24.1	11.1	71.1	56.9	15.0	9.6	59.1	24.9	11.3	18.0	66.1	18.3	13.5	21.1	9.9	10.1	15.3	22.0	9.7	36.6	151.3	7.4
VWS Equity	0.0%	20.0%	50.0%	80.0%	6.0%	14.9%	23.7%	6.1%	14.1%	22.1%	47.8%	2.8%	9.7%	33.0%	42.6%	24.4%	28.1%	31.2%	21.5%	42.0%	62.3%	81.5%	32.3%
TWS Equity	0.0%	20.0%	50.0%	80.0%	7.1%	17.9%	28.6%	9.2%	18.8%	28.4%	54.7%	6.2%	17.0%	41.8%	51.8%	29.8%	38.1%	44.5%	21.3%	41.0%	60.5%	79.0%	49.3%

Payout Option: Withdrawal Program + Deferred Annuity

RepRate	FP0	FP20	FP50	FP80	LD20	LD50	LD80	SF25	SF42.5	SF60	SF80	PLF20	PLF50	PLF80	PLF90	AMS50	AMS80	AMS100	DRB20	DRB40	DRB60	DRB80	DMS
CVaR-1%	20.4%	19.6%	14.0%	9.2%	20.6%	19.1%	16.8%	20.9%	19.9%	18.6%	11.6%	20.5%	18.3%	13.4%	11.7%	17.6%	16.4%	15.0%	19.4%	15.9%	12.2%	9.4%	10.8%
Q-1%	21.0%	21.0%	15.7%	11.0%	21.6%	20.7%	18.7%	21.8%	21.3%	20.2%	13.3%	21.3%	20.0%	15.0%	13.4%	19.4%	18.2%	16.8%	20.8%	17.5%	13.8%	11.0%	14.2%
CVaR-5%	21.6%	22.2%	17.7%	12.9%	22.5%	21.9%	20.0%	22.7%	22.5%	21.5%	15.2%	22.2%	21.1%	16.8%	15.2%	20.7%	19.6%	18.3%	22.1%	19.3%	15.9%	13.0%	17.2%
Q-5%	22.5%	24.1%	20.6%	16.0%	24.0%	23.8%	22.4%	24.0%	24.2%	23.6%	18.2%	23.6%	23.0%	19.6%	18.2%	23.0%	22.0%	20.9%	24.0%	22.1%	19.0%	16.0%	21.8%
Q-25%	25.2%	29.6%	30.7%	28.8%	28.1%	29.8%	30.3%	27.9%	29.5%	30.3%	29.6%	27.6%	29.6%	30.0%	29.4%	30.4%	30.4%	30.1%	29.6%	30.4%	29.8%	28.3%	32.1%
Median	28.0%	34.5%	40.6%	43.7%	31.9%	35.3%	38.1%	31.6%	34.3%	36.5%	42.0%	31.4%	35.7%	40.7%	41.9%	37.4%	38.9%	39.9%	34.7%	39.0%	42.0%	43.7%	40.8%
Mean	30.6%	36.7%	45.3%	55.8%	34.3%	37.7%	41.6%	34.1%	36.6%	39.1%	50.3%	33.8%	38.4%	47.1%	50.8%	40.4%	43.0%	45.7%	37.0%	42.7%	49.4%	56.6%	43.0%
Q-75%	32.4%	40.9%	54.9%	68.4%	37.2%	42.7%	48.8%	36.8%	40.8%	44.8%	61.4%	36.6%	43.9%	57.2%	61.8%	46.9%	50.7%	54.1%	41.6%	50.7%	60.1%	68.4%	51.7%
Q-95%	46.9%	56.0%	85.7%	136.6%	52.6%	59.2%	72.4%	52.2%	56.4%	63.6%	109.7%	51.9%	62.6%	95.7%	112.0%	67.6%	78.3%	90.8%	57.2%	76.4%	105.6%	140.4%	70.9%
Q-99%	71.9%	78.3%	116.8%	213.0%	77.0%	80.9%	96.2%	76.6%	79.5%	82.8%	165.8%	76.7%	83.3%	139.1%	172.5%	88.1%	105.7%	130.1%	80.3%	102.7%	153.4%	232.1%	88.3%
Volatility	11.0%	11.6%	21.4%	43.8%	11.4%	12.4%	16.5%	11.2%	11.6%	13.3%	32.2%	11.4%	13.4%	26.0%	33.5%	14.7%	18.6%	23.9%	11.9%	17.7%	29.8%	47.2%	15.8%
Skewness	7.5	3.3	1.8	3.4	5.4	2.7	1.8	5.3	3.2	2.1	2.6	5.5	2.4	2.2	2.8	1.9	1.9	2.4	3.1	1.7	2.5	4.0	1.2
Kurtosis	121.1	31.0	8.9	26.3	72.0	21.7	9.5	66.5	27.9	12.3	15.4	73.3	17.2	12.5	18.3	10.3	10.1	15.2	27.8	8.3	16.4	36.7	8.0
VWS Equity	0.0%	20.0%	50.0%	80.0%	7.4%	18.4%	29.3%	7.2%	15.8%	24.3%	56.7%	5.5%	18.4%	45.5%	55.3%	28.3%	33.8%	38.6%	21.6%	42.0%	62.3%	81.5%	33.7%
TWS Equity	0.0%	20.0%	50.0%	80.0%	10.0%	25.0%	40.0%	11.8%	22.7%	33.6%	63.9%	10.0%	27.7%	55.4%	65.4%	35.5%	47.7%	57.2%	20.7%	40.0%	59.0%	77.0%	52.0%