

# Financial Study

## Assessing the Nature of Investment Guarantees in Defined Contribution Pension Plans

A Background Paper to the OECD Policy Report DAF/AS/PEN/WD (2010)/12:  
“The Role of Guarantees in Defined Contribution Pensions”

Authors:

Dr. Gerhard Scheuenstuhl\*

Dr. Sandra Blome<sup>#</sup>

David Karim\*

Matthias Moch\*

Stefanie Brandt<sup>#</sup>

\* risklab GmbH

<sup>#</sup> Institute for Finance and Actuarial Sciences

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# 1 Introduction

Guarantees in the context of defined contribution (DC) pension plans play an increasingly important role in funded private pension and retirement systems. Especially after the experiences of recent financial market crises, the request for higher security and reliance on expected pension income has become more prominent. Guaranteed components in the context of pensions are useful not only from the perspective of the individual plan member, but also from a socio-political and welfare perspective.

Structurally, different types of guarantees are found in various forms and in many countries. In some countries, they are even mandatory within the pension and retirement system. In this context, the question that arises is how to assess the cost-benefit effects of guarantee components on retirement income and if there should be specific regulations for these kinds of pension plans.

This study performs a systematic analysis on the effect of introducing investment guarantees to DC savings plans and their effect on the future retirement income situation. It is aimed at providing a transparent basis for assessing the effect of different types of guarantees currently under discussion and helping DC plan designer and its plan members to make an informed decision on assessing their cost-benefit characteristics. The analysis also wants to provide qualitative input and numerical results to support discussions with supervisory bodies regarding efficient regulations for DC pension funds – especially for those plans being mandatory or offered as a default option. The outcome of this study should rather be understood as a detailed base of information for making decisions when considering the introduction of guarantee concepts. For any precise recommendation, the general framework of the funded private pension and retirement system has to be taken into account.

The analysis focuses on a standard DC pension plan regarding the level of contributions, underlying salary development and typical life cycle investment strategy to illustrate and quantify the basic characteristics. A detailed modeling of the underlying DC pension plan is introduced in Section 2. Sensitivity analyses are added to quantify effects on changing those basic plan characteristics and variations of financial market conditions to achieve further insight on the cost-benefit effects of these guarantees.

The investigated types of guarantees for this underlying DC plan are explained in Subsection 3.1. The study considers not only a set of structurally different types of guarantees, that can be found in existing pension regimes but also those, which are currently under discussion by experts. When introducing guarantees into DC plans the question concerning the additional costs in relation to a higher security level will arise. A major aspect of the analysis therefore concerns the pricing of such guarantees. In

Subsection 3.2, we calculate the “fair value” of a guarantee based on a risk neutral pricing framework, i.e. specify the amount of money needed to finance a given guarantee under the given pension plan. Considering this aspect the resulting guarantee fees are very sensitive regarding different financial market conditions, different savings phase horizons, different choices of investment strategies and also differences in the structure of deductions<sup>1</sup>. Sensitivity analyses regarding changes in those parameters are performed in Subsections 3.4, 4.3 and 5.3.

Analyzing the cost of a guarantee is a rather complex issue. In this study, we calculate the guarantee fees by using the theory of fair pricing under a generic financial market situation. Naturally, the conceptual framework being used has to abstract from some “practical world” features, which would also affect guarantee prices. To keep the analysis comprehensible and to reflect that practical circumstances are quite different in different jurisdictions and under different regulations (like solvency requirements) only a generic market situation will be modeled. Guarantee prices quoted in the market therefore might deviate from our results due to transaction cost, liquidity premiums or solvency capital requirements and could be significantly higher than the ones shown in this study.

The effect of providing guarantees for DC plan members will not only be analyzed in terms of guarantee fees but also based on its outcomes, i.e. the resulting retirement benefits. In Section 4, the benefits are measured in terms of potential retirement income provided in form of a life-long nominal annuity income stream, which can be bought at retirement age with the available final lump sum payment expressed in terms of a replacement rate. A comprehensive cost-benefit analysis of the introduced DC plan guarantees is provided. The results are based on an economic scenario analysis applying 10'000 Monte Carlo scenarios.

In Section 5, we extend the analysis evaluating guarantees from different angles, considering specific market stress scenarios, portability options and changes of guarantee design.

Finally, Section 6 concludes by summarizing the main results of our analyses.

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<sup>1</sup> There are various ways to pay the provider of a guarantee, which differently affect the results of the guaranteed pension outcome. For example the fees can be deducted from incoming contributions or from the so far accumulated net asset value of the savings account. Also the timing of the guarantee payments are relevant, i.e. guarantee fees may be paid evenly over the savings phase or are paid mainly in the beginning of the savings phase or rather at the end and even being conditional on the level of the resulting surplus.

## 2 Projection Model for DC Pension Plans

The underlying DC pension plan structure builds the basis of the following analysis. Considering the large number of variations in DC plan design, this study focuses on one typical plan structure, which can be found in many DC pension markets. In a first step, these basic characteristics of the plan are specified and modeled. The available asset universe and the plan's underlying investment strategy during the savings period are respectively described in Subsections 2.2 and 2.3.

### 2.1 Characteristics of the DC Plan

A defined contribution plan can be seen as a savings plan where the employer and the plan member contribute regular payments depending on the current salary to an individual retirement account. Unlike a defined benefit (DB) plan, a DC plan does not promise a specific amount of benefits at retirement but the plan member ultimately receives the balance in this account as a lump sum or in many cases an annuity is purchased with this balance at the time of retirement. Employees enrolled in a DC pension plan are therefore predominantly exposed to investment risk, inflation risk and longevity risk. In order to restrict the complexity of the model, we focus on investment and inflation risk during the savings phase.

For the specification of the underlying DC plan, we assume the following common characteristics:

- **Wage Process and Contribution Rates:**

We assume a contribution rate of 10% of the current wage. The underlying wage process depends on the one hand on inflation and on the other hand on the career and productivity dynamics. The last two factors typically are related to the age of the person and their modeling is based on the Panel Study of Income Dynamics<sup>2</sup> (PSID polynomial). Figure 1 illustrates the assumed wage dynamics.

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<sup>2</sup> J. Cocco, F. Gomes, and P. Maenhout: "Consumption and Portfolio Choice over the Life-cycle". Review of Financial Studies, Vol. 18 (2005).

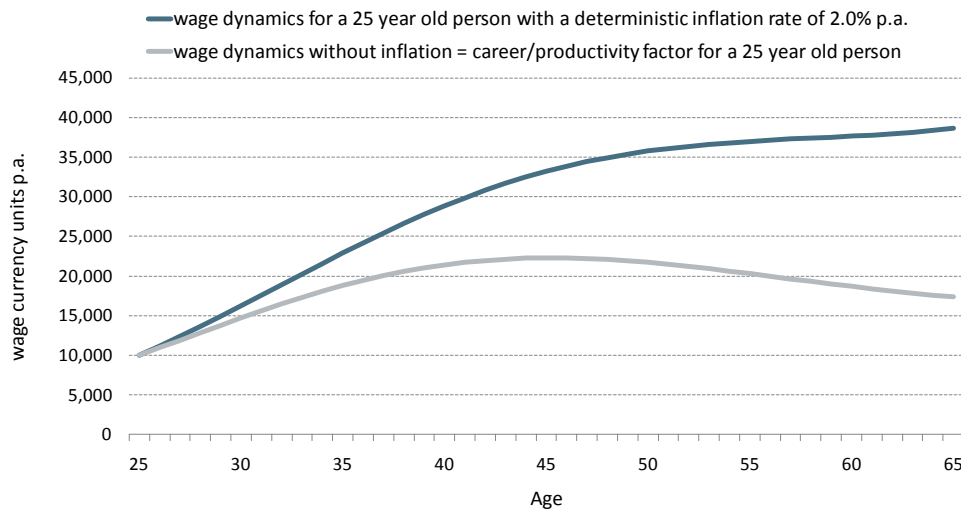


Figure 1: Illustration of the wage dynamics with a fixed inflation rate

For simplicity reasons, we set the initial wage to 10'000 currency units and its dynamics is described as following:

$$wage_t = \begin{cases} 10'000 & t = t_0 = 25 \\ wage_{t-1} \times (1 + inflation_t) \times career/productivity\ factor_t, & \forall 25 < t < 65 \end{cases}$$

The modeling of an average career and productivity dynamics is assumed to be independent of financial market developments. However, as inflation is part of the capital market model, we get different developments of the wage process depending on the stochastic influence of inflation and thus also a link to economic development. In Figure 2, the corresponding empirical probability distribution of the last wage before retirement is displayed. The broad range of possible results indicates that inflation risk during the savings period of 40 years has a substantial impact on final wage levels – even given a deterministic career path. Assuming additional uncertainty in carrier development would add some further risk to the results.

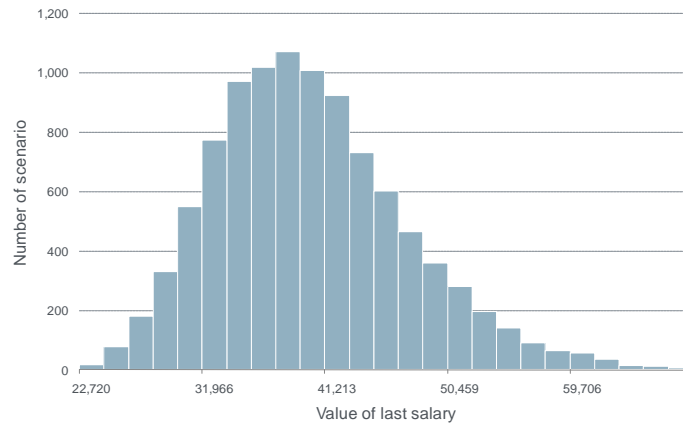


Figure 2: Histogram distribution of last wage before retirement

Corresponding to a stochastic wage development, the probability distribution of the contributions paid into the savings plan can be derived. Figure 3 displays this range of sums of cash-in-flows. Under a nominal capital protection, this sum will be the required minimum lump sum available for retirement income during the decumulation phase of the DC plan.

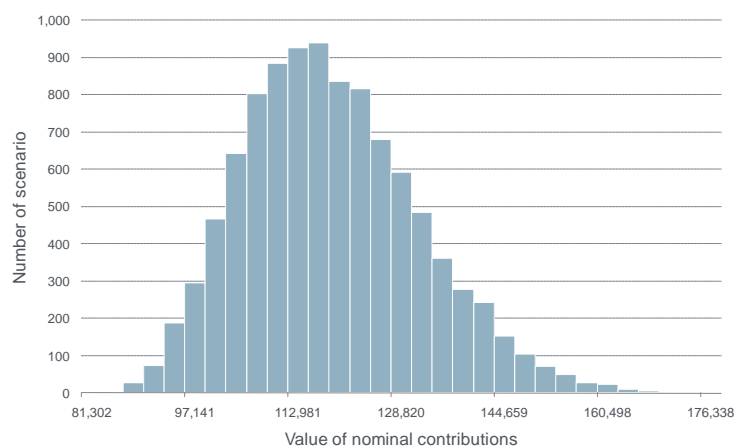


Figure 3: Histogram nominal contributions paid to savings plan

- **Investment Plan, Savings Period and Asset Universe:**

The contributions are paid during the whole savings phase of 40 years (assuming that an employee enters the plan at age 25 and reaches retirement at age 65). We assume that a person does not end the plan during this savings phase, neither by lapse nor by death. All contributions are invested according to an investment strategy, which will be described in Section 2.3. Any guarantees that might be part of the plan are described in Subsection 3.1. In our model, we abstract from administration fees charged by the plan provider or other operational transaction costs.

Regarding the benefit payments, our analysis focuses either on the lump sum at retirement or the replacement rate assuming a life-long annuity<sup>3</sup>. The life-long annuity is purchased at age 65 such that the lump sum accumulated is used as a single premium for an immediate life annuity. The benefit payments are considered being constant until death. The calculation of the annuity takes into account the term structure of the interest rates at age 65 and a standard mortality table<sup>4</sup>. No administration fees are being assumed for the annuity calculation. For a given scenario (i.e. for a given lump sum and a term structure of interest rates), the annuity payments are calculated as follows:

$$\text{Payments} = \text{Lump Sum} / \sum_{t=65}^{115} p_{65} \cdot (1 + r_{65})^{-t+65}$$

where  $p_{65}$  is the probability that a person aged 65 stays alive for at least  $t$  years and  $r_{65}$  is the term structure of interest rates at age 65.

The level of retirement income is often measured in relation to the last income before retirement. This relation is defined as a replacement rate. Based on the different 10'000 scenarios, we can derive a corresponding empirical distribution of replacement rates:  $\forall s \in \{1, \dots, 10000\}$ ,

$$\text{RepRate}_s = \frac{\text{Payments}_s}{\text{wage}_{s,65}}$$

where the payments and the wage at age 65 depend on the given scenario.

## 2.2 Asset Universe and the Stochastics of Financial Instruments

The asset class universe defines the basic types of financial instruments in which the contributions of the DC plan can be invested. This analysis focuses on the main asset classes that represent the traditional spectrum most pension plans invest their money in. The universe considered in this study comprises the following main asset classes:<sup>5</sup>

- Equity: A broadly diversified portfolio of international stocks like an index portfolio on the MSCI world equity index is assumed.<sup>6</sup>

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<sup>3</sup> Further payout options are described in P. Antolin: "Policy Options for the Payout Phase", OECD Working Papers on Insurance and Private Pensions, No. 25 (2008).

<sup>4</sup> The mortality table RT Heubeck 2005 G for occupational pensions in Germany is used, where 115 is the maximal age. (cf. K. Heubeck, R. Herrmann, and G. D'Souza, "Die Richttafeln 2005 G – Modell, Herleitung, Formeln", Blätter der DGVM, XXVII (3) (2006).)

<sup>5</sup> Note: Not explicitly considered are alternative asset classes such as real estate, commodities, private equity or hedge funds nor specialized investment such as infrastructure. Investments in these asset classes will be found in many real DC investment plans. For the sake of simplicity, however, they will not be included in this analysis. On the bond side, we focus only on government bonds, which are assumed to be safe investments.

<sup>6</sup> In the following discussions the risk of a portfolio is often associated with its exposure to equity positions.



- Government bonds: A broad portfolio of government bonds with different maturities is considered. The portfolio duration is 5.6 years.

Considering a multi-period savings phase, it is necessary to quantify the stochastic return behavior of the underlying financial instruments in order to model possible outcomes of the savings phase. In our analysis the risklab Economic Scenario Generator (ESG) is used as the conceptual modeling framework.<sup>7</sup> It incorporates fundamental macroeconomic factors (such as GDP or inflation) to describe the evolution of interest rates, credit spreads and (influenced by these) the return development of bond and equity portfolio positions among other alternative asset classes. Using a cascading structure, it captures the long term structural economic relationship between e.g. interest rates and inflation while allowing for short term stochastic deviations. This setting allows for an integrated modeling of financial markets, delivering economically meaningful and consistent scenarios. In order to build up a framework, 10'000 scenarios are modeled to represent the investment return of each asset class used in this study as well as inflation.

The model calibration requires the input of a set of parameters. Specific assumptions like long term expectations and volatility are used to model macroeconomic variables or the term structure of interest rates (used to model government bonds) among others. We derive forward looking assumptions from historical data and set the mean reversion levels of interest rates and inflation with possible fluctuations. The scenarios used in this study reflect a general economy and do not represent a specific country. A basic characterization of the asset classes and characteristics of real world scenarios are given in Appendix 7.1.3.

## 2.3 Investment Strategy

Life cycle investment strategies are often applied for DC plans.<sup>8</sup> This analysis assumes also a basic form of a life-cycle strategy. A common characteristic of these investment strategies is that the investment risk resulting from the risky asset positions (i.e. prevalently the equity position) is being reduced during the last years before retirement. In our case, the strategy assumes an asset allocation of 80% in equity from the beginning of the savings phase (here from age 25 to age 55) followed by a linear reduction of the equity position until the end of the savings period to reach an allocation of 20% in the risky asset at retirement age.

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<sup>7</sup> For details see Appendix 7.1.2.

<sup>8</sup> An extensive overview and characterization of those strategies can be found in G. Scheuenstuhl, S. Blome, W. Mader, D. Karim, and T. Friederich: "Assessing Investment Strategies in Defined Contribution Pension Plans under Various Payout Options" (2010), risklab working paper series under:

<http://www.risklab.com/de/meta/publikationen/index.html>

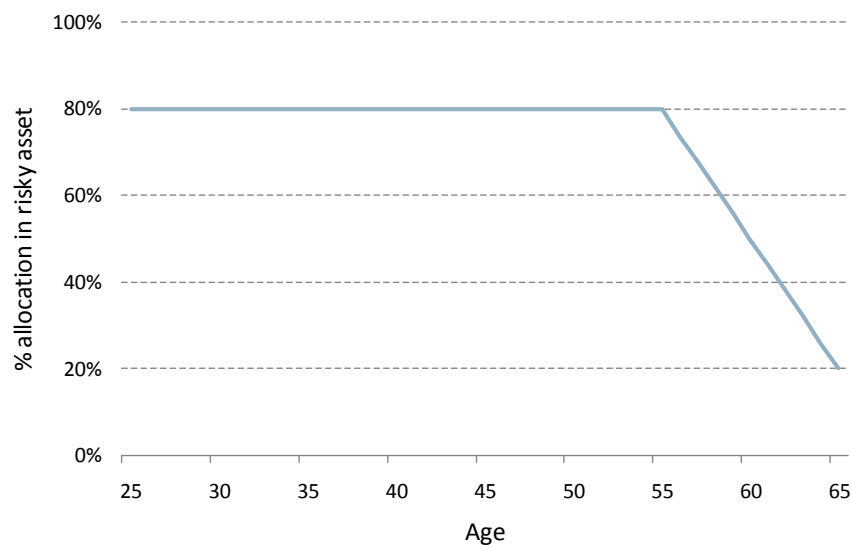


Figure 4: Equity allocation of life cycle strategy during the savings phase

The chosen risk profile of the underlying investment strategy has obviously a relevant impact on the outcome of the guarantee analysis. To assess the impact of this special shape, we will also consider modifications of the equity exposure and glide path.

### 3 Modeling Investment Guarantees

Guarantees are found in various forms and context. This analysis focuses on some basic types of structurally different guarantees on the result of capital accumulated during or at the end of the saving period.

#### 3.1 Guarantees Analyzed

In this subsection a short characterization of the six types of guarantees used in the further analysis will be given. For each scenario  $s$  the guarantee is defined as follows:

##### Guarantee G\_0:

It guarantees that the lump sum at retirement equals at least the nominal sum of contributions made,<sup>9</sup> which can be written as:

$$\sum_{t=1}^T [\text{Contribution}(t, s)]$$

<sup>9</sup> This type of guarantee can be found for example in Germany in both the occupational DB pension plans as well in the so-called Riester products.

**Guarantee G\_real:**

The corresponding guarantee in real terms is that the lump sum at retirement equals at least the sum of contributions in real terms, i.e. this is a guaranteed real interest rate of 0%:<sup>10</sup>

$$\sum_{t=1}^T \left[ \text{Contribution}(t, s) \cdot \prod_{l=1}^t (1 + \text{Inflation}(l, s)) \right]$$

Note that this is a scenario dependent guarantee, where the inflation rate in each scenario and in each point of time is applied when the contributions are paid.

**Guarantee G\_2:**

As the average inflation rate in our model is about 2%, we also define the following guarantee, which provides a similar level of guarantee as G\_real. Here the lump sum at retirement age equals at least the amount generated when all contributions grow with a guaranteed nominal interest rate of 2% p.a. over the whole investment horizon:<sup>11</sup>

$$\sum_{t=1}^T \left[ \text{Contribution}(t, s) \cdot 1.02^{T-t+1} \right]$$

**Guarantee G\_4:**

Concerning the guarantee level, we observe low levels (mostly in combination with higher upside potential) and also higher guarantee level aiming for a high security level (but often with lower upside potential). G\_4 guarantees a lump sum at retirement age that equals at least the amount generated when all contributions grow with a guaranteed nominal interest rate of 4% p.a. over the whole investment horizon. This defines a relatively high minimum return guarantee:

$$\sum_{t=1}^T \left[ \text{Contribution}(t, s) \cdot 1.04^{T-t+1} \right]$$

**Guarantee G\_float:**

The guarantees defined above provide a guarantee level, which can be determined in advance for fixed contribution payments (apart from G\_real). However, as the capital market might change significantly during the savings

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<sup>10</sup> For instance, both public and private pension funds in Uruguay have to provide a guarantee which is based on real terms. For details cf. J.A. Turner and D.M. Rajnes: "Rate of return guarantees for mandatory defined contributions plans", International Social Security Review, Vol. 54, 4/2001, p. 57.

<sup>11</sup> Literature refers to this type of guarantees as "minimum return guarantees" (e.g. J.Turner: "The design of rate of return guarantees for defined contribution plans", Journal of Pensions Management Vol. 7,(2001))

phase these types of guarantees might become for instance very expensive from the pension fund's view (e.g. low interest rates) or less valuable for the plan member (e.g. high interest rates).  $G_{float}$  provides a type of guarantee, where the guaranteed interest rate depends on the current capital market when a contribution is paid.<sup>12</sup> This floating guaranteed interest rate is defined as the current 1-year interest rate at time  $t$ . The assumed rate is assigned to each contribution made and is valid until retirement in each scenario. Hence, at each point of time and in each scenario we have a different guaranteed interest rate. This implies a high guarantee in case of high 1-year rates at time of contribution payment and a low guarantee in case of low 1-year rates.<sup>13</sup>

$$\sum_{t=1}^T [Contribution(t, s) \cdot (1 + GovRate1(t, s))^{T-t+1}]$$

#### Guarantee $G_{ongoing}$ :

Most guarantees are valid at the age of retirement only. In case a plan member changes his pension plan by transferring the value of accumulated assets to another plan, the guarantee does not hold. To cover portability aspects,  $G_{ongoing}$  requires that at each point of time (on an annual basis) the accumulated assets equal at least the sum of contributions made until then. The guarantee holds during the whole savings phase and is therefore much more valuable than the other guarantees, which are valid at retirement only.

$$\sum_{t=1}^{\tau} [Contribution(t, s)] \quad \forall \tau \in \{1, \dots, T\}$$

### 3.2 Fair Pricing of Guarantees

In general, fair pricing of a guarantee means finding the cost for which the present value of the expected future guarantee fees equals the present value of the expected future guarantee claims. But as guarantee fees and guarantee claims depend on the development of the contributions and the underlying investment strategy a risk-neutral model is necessary to determine a value of such a guarantee.<sup>14</sup>

Under certain conditions of the model, closed form solutions exist for the prices of financial derivatives, e.g. the Black-Scholes formula for vanilla options. For more complex

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<sup>12</sup> A similar system can be found in Denmark (ATP).

<sup>13</sup> Literature also refers to these floating guarantees as "relative" (e.g. Turner/Rajnes).

<sup>14</sup> For the pricing of options see for example J. Cox and S. Ross 1976, "The Valuation of Options for Alternative Stochastic Processes", Journal of Financial Economics 3, p.145-166.

capital market models and guarantees, no closed form solutions exist and simulations are needed to determine the costs.<sup>15</sup>

Details on the model used for the valuation of the fair price of the types of guarantees specified in the previous section can be found in Appendix 7.1.1.

Without any guarantee the lump sum at retirement ( $LS(T)$ ) equals the net asset value (NAV) of all contributions invested in the life cycle strategy.

$$LS(T) = NAV^{LifeCycle}(T)$$

with:

$$NAV^{LifeCycle}(t) = NAV^{LifeCycle}(t-1) \cdot (1 + \text{Return}^{LifeCycle}(t-1, t)) + \text{Contribution}(t).$$

By including for example a nominal guarantee structure in the investment plan, the lump sum equals at least the sum of the contributions:<sup>16</sup>

$$LS(T) = \max\left(NAV^{LifeCycle}(T), \sum_{t=1}^T \text{Contribution}(t)\right)$$

To be able to determine a price of this risk reduction, we can decompose the lump sum into the performance of the assets and an optional component:

$$LS(T) = NAV^{LifeCycle}(T) + \max\left(0, \sum_{t=1}^T \text{Contribution}(t) - NAV^{LifeCycle}(T)\right)$$

There are several possibilities to pay for this guarantee feature:

- Regular payment as a percentage of the accumulated net asset value of all contributions invested into the life cycle strategy.
- Regular payment as a percentage of every contribution.
- Upfront payment.
- Selling upside potential.

From a practical perspective, the first two possibilities are the most meaningful ones. The following analysis will focus on a payment as a percentage of the NAV:

$$NAV^{LifeCycle}(t) = \left(NAV^{LifeCycle}(t-1) \cdot (1 + \text{Return}^{LifeCycle}(t-1, t)) + \text{Contribution}(t)\right) \cdot (1 - \text{guarantee costs})$$

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<sup>15</sup> Note that risk neutral simulations are used for the valuation of the fair price of a guarantee and real world simulations are used for projecting the DC plan. Details can be found in the Appendix 7.1.

<sup>16</sup> Note that the NAV does not include any guarantee fees at this stage.

After a simulation of the risk factors, scenarios for the contributions and the investment strategy can be generated that depend on the risk factors of the model:

$$NAV^{LifeCycle} = NAV^{LifeCycle}(r, S, i)$$

From the scenarios of the NAV the costs of a guarantee can be deduced for every scenario as well as the payoff of the guarantee that depends on the value of the NAV at time T.

The final costs for the guarantees are then determined such that the following equation holds:

$$E[PV(\text{costs}(\text{guarantee}))] = E[PV(\text{payoff}(\text{guarantee}))]$$

### 3.3 Guarantee Fees

In this study as mentioned above, we calculate the guarantee fees by using the theory of fair pricing under a generic financial market situation. Naturally, the conceptual framework being used has to abstract from some “practical world” features, which would also affect guarantee prices. To keep the analysis comprehensible and to reflect that practical circumstances are quite different in different jurisdictions and under different regulations (like solvency requirements) only a generic market situation will be modeled. Guarantee prices quoted in the market therefore might deviate from our results and due to transaction cost, liquidity premiums or solvency capital requirements, they could be significantly higher than the ones shown in this study.

On the basis of the generated scenarios, the expected values can be numerically calculated using Monte Carlo simulation and evaluated for different guarantee costs. Then a numerical optimization routine is designed to find the costs for the guarantee, such that the equation holds. The following table summarizes these costs for the different guarantees under consideration:

	G_0	G_real	G_ongoing	G_2	G_4	G_float
% of NAV	0.055%	0.241%	0.389%	0.218%	0.887%	1.224%
% of Contribution	1.244%	5.583%	18.364%	4.936%	18.709%	26.088%
Guaranteed benefits	118,160	171,895	118,160	169,438	252,013	261,768

Table 1: Comparison of guarantee fees

The costs for the different types of guarantees are obviously quite different. The following reasoning can be given to motivate these differences:

- Guarantee fees as a percentage of each contribution are much higher compared to a percentage of the NAV, since the NAV grows much more over time.
- The fees for the guarantees G\_real and G\_2 are similar as they deliver similar guarantee levels in average.<sup>17</sup> G\_2 is the cheaper guarantee as the nominal rate of return is fixed in advance. Guarantee G\_real is more expensive, as it includes inflation protection.
- The guarantee G\_0, which guarantees at least the contributions paid, is the cheapest since it does not pay a guaranteed interest on the contributions.
- G\_ongoing provides also a guarantee level of 0%. However, the guarantee fees are very high since the guarantee is valid during the whole savings phase.
- The fees for guarantee G\_4 and G\_float are very high compared to the others, as their guarantee levels are high.

### 3.4 Sensitivities of Guarantee Fees

The results shown in Table 1 provide a good first assessment of the different guarantee costs. However, as pointed out above these fees were derived based on a specific set of assumptions and model parameters. A deeper analysis is necessary to understand the impact of our model settings and market risk factors assumptions on the pricing of these different guarantees. This enables us to get a better understanding of the possible range of costs of such guarantees and what factors mostly affect them.

In Subsection 3.4.1, we show the effects in case the capital market environment changes. In the next subsection, we will look at the effects resulting from the length of the assumed investment horizon. How will guarantee fees change if the savings phase is reduced from a period of 40 to 20 years? Finally, in Subsection 3.4.3 we show the results on guarantee fees, if the underlying investment strategy is modified.

Additional analyses are provided in Appendix 7.2, where we show the sensitivity of guarantee price depending on the market scenario by presenting results assuming conditions of specific points of time in the past.

#### 3.4.1 Fee Effects Resulting from Changes in Capital Market Environment

The price of the underlying guarantee depends on financial market parameters such as volatilities, interest rates and inflation rates. These financial market parameters change

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<sup>17</sup> The “Guaranteed benefits” described in the table is the average value of the guarantee level distribution.

over time. So plan members entering the DC plan at different times will face different conditions for their guarantees. For example in a high volatile market scenario, guarantee fees are much higher compared to a calm market scenario as the risk of falling short the guarantee level is much higher. The following table summarizes the fees for the different types of guarantees:

	G_0	G_real	G_ongoing	G_2	G_4	G_float
Fee Base Parameters (% of NAV)	0.055%	0.241%	0.389%	0.218%	0.887%	1.224%
parallel shift of +10% of volatility term structure	0.1078% (+96%)	0.37837% (+57%)	0.53682% (+38%)	0.34662% (+59%)	1.1531% (+30%)	1.70136% (+39%)
parallel shift of -1% of interest rate term structure	0.109% (+99%)	0.472% (+96%)	0.541% (+39%)	0.423% (+94%)	1.801% (+103%)	1.236% (+1%)
parallel shift of + 1% inflation term structure	0.057% (+4%)	0.472% (+96%)	0.393% (+1%)	0.225% (+3%)	0.905% (+2%)	1.285% (+5%)

Table 2: Fee effects for the different types of guarantees due to changes in capital market environment

As expected the guarantee fee of G\_real is very sensitive to an inflation term structure change and the guarantee fee of G\_float is not very sensitive to an interest rate term structure change. All guarantee fees rise if volatilities increase.

### 3.4.2 Fee Effects Resulting from Changing the Length of the Investment Period

A change of the investment period will have a significant impact on the level of guarantee fees since the sum of contributions will change – naturally also on the level of guaranteed benefits.

The main effect will be that guarantees fees will increase when the savings period becomes shorter. For a 20 years investment period the volatility risk has more impact than for a 40 years investment period, since the drift has more impact than the volatility risk for the longer period. The NAV is also smaller on average, the shorter the investment period gets.

Besides we observe a smaller guarantee fee for G\_float than for G\_4 in case of a 20 year investment horizon, although guaranteed benefits are quite similar. When considering the 40 year investment horizon this was the opposite. This is due to the risk of rising interest rates over longer periods.



40 years	G_0	G_real	G_ongoing	G_2	G_4	G_float
% of NAV	0.055%	0.241%	0.389%	0.218%	0.887%	1.224%
Guaranteed benefits	118,160	171,895	118,160	169,438	252,013	261,768

20 years	G_0	G_real	G_ongoing	G_2	G_4	G_float
% of NAV	0.240%	0.843%	0.913%	0.886%	4.036%	3.321%
Guaranteed benefits	43,959	52,764	43,959	52,443	63,074	62,555

Table 3: Comparison of the guarantee fees for a 40 years and a 20 years savings period

### 3.4.3 Fee Effects Resulting from Changing the Underlying Investment Strategy

The life cycle investment strategy underlying the previous guarantee pricing assumed a start allocation in equity of 80% (LC 80), cf. Subsection 2.3. Two other life cycle strategies are introduced in order to assess whether the risk induced by the equity exposure has an impact on the different guarantees.

LC 50 is a strategy, which involves a 50% allocation in equity from age 25 to age 60 followed by a linear reduction of the equity position to reach a 20% allocation in the risky asset at retirement age. LC 20 is a strategy, which involves a constant allocation of 20% in equity from age 25 to the retirement age.

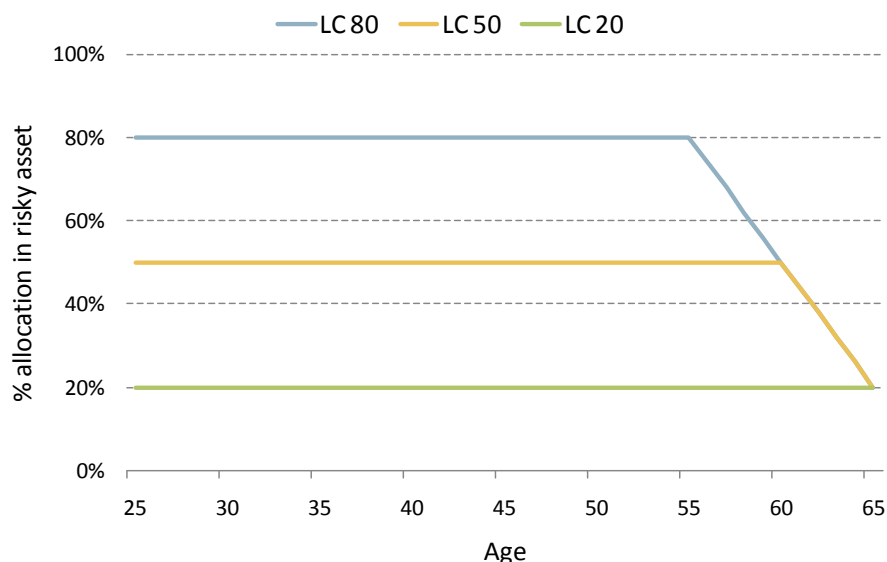


Figure 5: Introducing two alternative equity glide paths for the life cycle strategy

Concerning the fees associated to the different guarantees, a reduction can be observed as the allocation in equity decreases. The level of guarantee within one given guarantee design is the same since it is independent of the equity allocation. Hence for a given

guarantee, the fees are going to be higher the higher the equity exposure (and vice versa), as shown in the following table:

	G_0	G_real	G_ongoing	G_2	G_4	G_float
% of NAV (LC 80)	0.055%	0.241%	0.389%	0.218%	0.887%	1.224%
% of NAV (LC 50)	0.027%	0.154%	0.146%	0.138%	0.705%	0.900%
% of NAV (LC 20)	0.007%	0.071%	0.020%	0.062%	0.490%	0.441%
Guaranteed benefits	118,160	171,895	118,160	169,438	252,013	261,768

Table 4: Comparison of the guarantee fees given different life cycle strategies

If equity exposure is replaced by bond exposure G\_float would be cheaper than G\_4 as the bond investments already address a part of interest rate risks. Furthermore G\_ongoing is very sensitive to the equity exposure as the investment strategy's volatility is the main factor of the guarantee fee.

## 4 Assessing Effects of Guarantees on Retirement Income

Characterizing guarantees by comparing only guarantee fees and the mean of guaranteed benefits is not sufficient to assess their full cost-benefit profile within a DC plan. In this section, we will address the effect of the different guarantees on the ultimate purpose of considering guarantees in the first place: the risk profile of the outcome of the retirement income distribution for plan members and an individual cost-benefit assessment.

As the comparison of the different guarantees is a highly complex issue, we extend the analysis in Section 5 by evaluating the guarantees from different perspectives, which is strictly necessary to understand the specific characteristics of the different types of guarantees. Both the results of Section 4 and Section 5 have to be taken into consideration. The main conclusions are written in Section 6.

### 4.1 Measures of Risk and Return for Retirement Income

In order to describe the characteristics of this retirement income given a guarantee, the empirical distribution of replacement rates is plotted and the corresponding statistical values (such as e.g. mean and standard deviation) are compared. In addition a set of risk measures, expressing different views on the assessment of the retirement situation, will be introduced and discussed.

#### 4.1.1 Measures of Risk for Retirement Income

One of the main concerns of retirees is that their retirement income might be very low or even below a critical level. There are different ways of assessing the downside potential of a replacement rate. The two following measures of risk deal with this issue from different points of view:

- **Value at Risk** of the replacement rate distribution (on a 95% confidence level):

This risk measure describes the result that could happen under very unfavorable circumstances. The measure represents the highest replacement rate value achieved by the 500 worst scenarios. Thus, in 95% of the scenarios, the replacement rate values are higher than this risk level. This risk measure is directly computed by identifying the 5% percentile value of the empirical replacement rate (RepRate) distribution.

$$\text{VaR}_{5\%} = \inf \{x, P(\text{RepRate} < x) \geq 5\%$$

Note that the higher the “Value at Risk” figure is, the lower is the risk of low replacement rates. Other confidence levels can also be used as e.g. 99%. Corresponding to the downside risk measure, one can use the value at risk concept also to assess the upside potential, e.g. by calculating  $\text{VaR}_{95\%}$  or  $\text{VaR}_{99\%}$ .

- **Conditional Value at Risk** of the replacement rate distribution on a 95% confidence level (which gives the expected replacement rate in the 5% worst cases):

Given the definition of the empirical replacement rate distribution, a high CVaR(5%) is better than a lower CVaR(5%).

$$\text{CVaR}_{5\%} = E[\text{RepRate} | \text{RepRate} < x], \text{ where } x \text{ is given by } \text{VaR}_{5\%}$$

Note:  $\text{CVaR}_{5\%} \leq \text{VaR}_{5\%}$ . The CVaR can also be used as a measure for upside potential.

Other risk measures can provide further information on the inherent risk of a pension plan. The probability that the lump sum from the DC plan will be even less than the amount of contributions paid in during the savings phase is a risk measure, which is rather easy to understand from a plan member’s point of view.

- **Probability that lump sum is below the sum of the contributions in real terms:**

$$I_s = \begin{cases} 1 & \text{if } \text{Lump Sum}_s < \sum_{t=25}^{65} \text{Contribution}_{s,t} \times \text{InflationIndex}_{s,64}, \forall s \in \{1, \dots, 10000\} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Probability} = \frac{1}{10000} \times \sum_{s=1}^{10000} I_s$$

- **Probability that the net asset value (NAV) is below the sum of the contributions paid at a given point of time t during the savings phase:**

$$I_s = \begin{cases} 1 & \text{if } NAV_{s,t} < \sum_{l=25}^t \text{Contribution}_{s,l}, \forall s \in \{1, \dots, 10000\} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Probability} = \frac{1}{10000} \times \sum_{s=1}^{10000} I_s$$

- Associated to the previous probability, an **expected shortfall** can be calculated i.e. the average value of the difference between the NAV and the sum of the contributions to that date, in case the NAV is below the sum of contributions paid.

#### 4.1.2 Measures of Return for Retirement Income

The average level of replacement rates is a major orientation to assess the quality of a DC plan. To characterize the average outcome of a distribution of scenario results we will use the statistics of median or mean:<sup>18</sup>

- **Median of the replacement rate distribution:** This measure represents an average replacement rate the retiree can expect to achieve with a DC pension plan. For the median, 50% of the scenarios are above and 50% of them are below this value.

This return measure is directly computed by identifying the 50% percentile value of the replacement rate distribution.

$$\text{Median} = \inf \{x, P(\text{RepRate} < x) \geq 50\%$$

- Similarly, the median value of the lump sum distribution will be used.

In the following figure, some of the risk and return measures presented above are summarized.

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<sup>18</sup> In this analysis we focus on the median value. Under asymmetric replacement rate distributions it provides the more conservative assessment of average income levels.

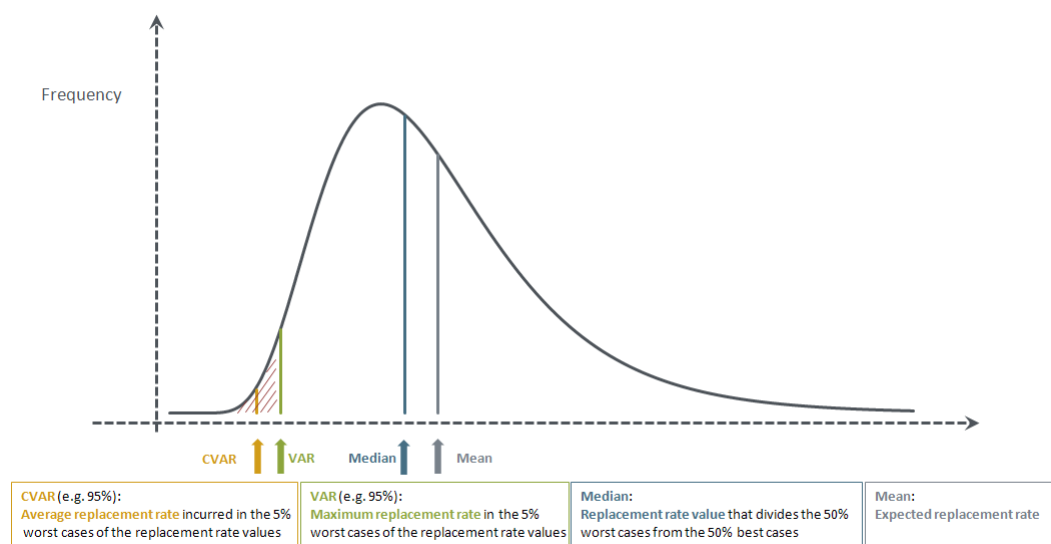


Figure 6: Description of the risk return measures based on the replacement rate distribution

## 4.2 Guarantee Effects on Retirement Income Distribution

In this subsection we summarize the statistical results of the real world simulation analysis – given the basic DC plan assumption. We focus on plan member’s view by analyzing the potential benefit payments. If not mentioned, all data shown in the following are based on guarantee fees, which are calculated based on the NAV.

For a simplified representation of the various guarantees within the result diagrams, we will use shorthand logograms with specific colors for the different guarantees. The following table summarizes these abbreviations with the corresponding description. As a benchmark, we analyzed a pension plan where no guarantees are provided. In the following we refer to this plan as no\_G.

1	no_G	No consideration of any guarantee concept
2	G_0	Guaranteed nominal interest rate of 0%
3	G_2	Guaranteed nominal interest rate of 2%
4	G_4	Guaranteed nominal interest rate of 4%
5	G_real	Guaranteed real interest rate of 0%
6	G_ongoing	Annual based guarantee
7	G_float	A guaranteed interest rate is assigned to each contribution made and is valid until retirement

Table 5: Overview shorthand logograms for investigated guarantees

For the following analyses we use risk return diagrams. The risk dimension is plotted on the abscissa, the return dimension on the ordinate. The more to the right a guarantee is located in the diagram the more risky it is. Both risk and return measures vary during the analyses.

In addition, for each guarantee a table with statistical data on the replacement rate distributions is given. In the first part of this table, the value at risk for different confidence levels is stated with the corresponding percentile (e.g. 5%-Q meaning  $Var_{5\%}$ ). In the middle part different CVARs are given and at the bottom the mean and the standard deviation (STD).

#### 4.2.1 Nominal (G\_0) and Real (G\_real)

A guarantee can either be expressed in nominal or in real terms. In this subsection, we compare two guarantees with a minimum return guaranteed of 0% in nominal terms (G\_0) and real terms (G\_real).

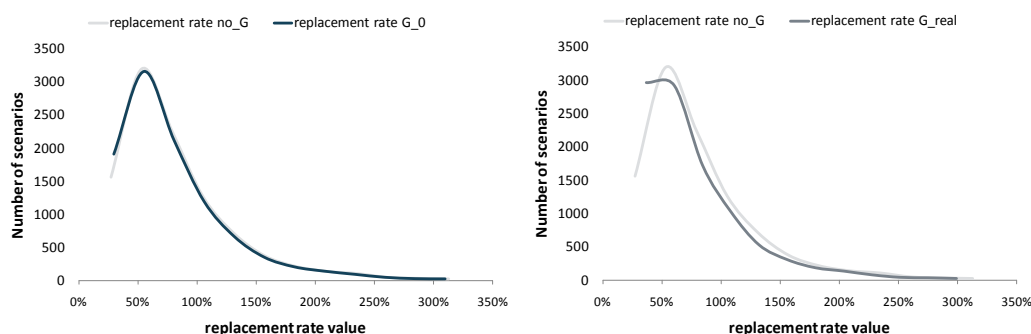


Figure 7: Empirical distributions of replacement rates G\_0 and G\_real

G\_0 appears to be very similar compared to the plan without any guarantees. Regarding G\_real the risk is of course smaller (cf. also VAR and CVAR for 1% and 5% resp. in the table below). On the other hand the upside potential for G\_real is lower, which is shown in the figure above as well as in the VAR and CVAR for 95% and 99%.

	Replacement Rate		
	no_G	G_0	G_real
	LC 80	LC 80	LC 80
1%-Q	22.65%	22.62%	27.85%
5%-Q	29.97%	29.66%	30.87%
25%-Q	47.72%	47.15%	45.26%
50%-Q	68.35%	67.49%	64.63%
75%-Q	100.39%	99.01%	94.60%
95%-Q	184.19%	181.51%	172.39%
99%-Q	287.82%	283.44%	269.32%
1%-CVaR	20.01%	20.72%	26.99%
5%-CVaR	25.56%	25.51%	29.04%
95%-CVaR	73.73%	72.77%	69.89%
99%-CVaR	79.64%	78.58%	75.38%
Mean	82.62%	81.52%	78.15%
STD	54.93%	54.04%	50.91%

Table 6: Statistical data for G\_0 and G\_real

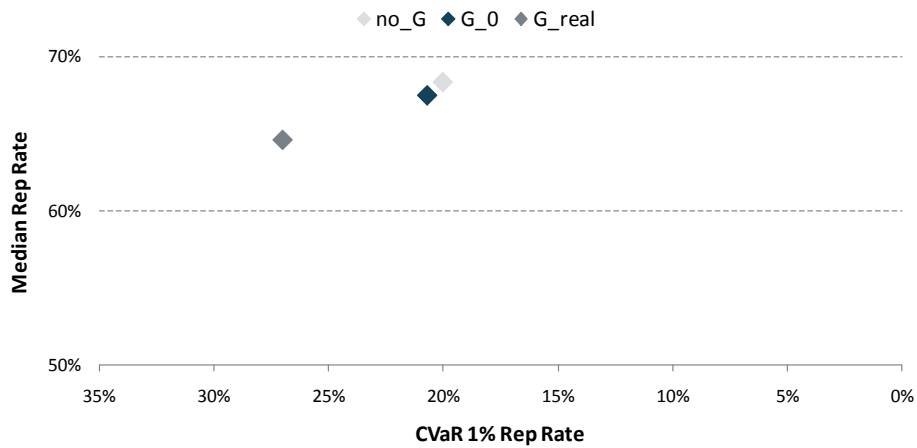


Figure 8: Risk return profile 1 for G\_0 and G\_real

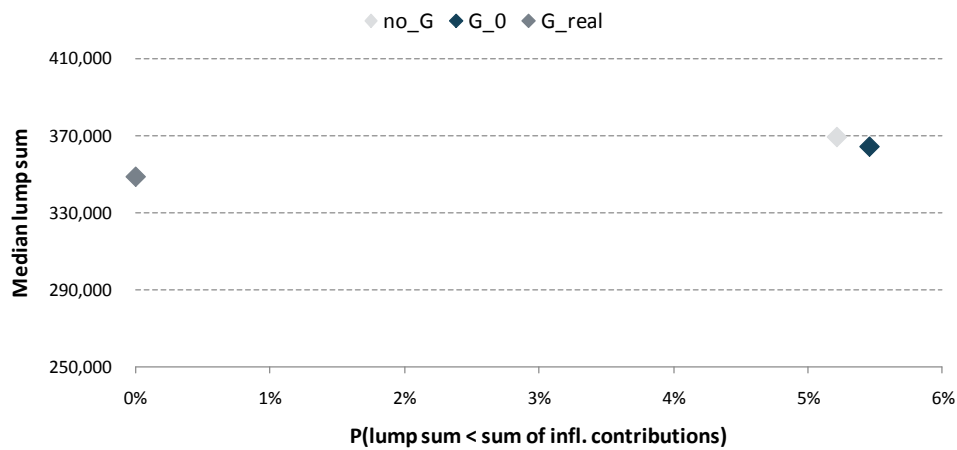


Figure 9: Risk return profile 2 for G\_0 and G\_real

Also both risk return profiles emphasize that G\_0 and no\_G are very similar in this setting. However, in risk return profile 2 (where the risk is the probability that the lump sum is below the sum of contributions in real terms) no\_G slightly dominates G\_0. Regarding G\_real, we observe in both profiles far less risk but also less return compared to G\_0. When looking at the second risk return profile, G\_real has by definition no risk. Hence comparing a nominal rate guarantee with a real rate guarantee, the choice would depend on the risk being focused and could lead to a trade-off between risk and return as measured by the replacement rate.

#### 4.2.2 At Retirement only (G\_0) and Ongoing during the Savings Phase (G\_ongoing)

In this subsection, we compare two guarantees, which both provide a nominal guarantee level of 0%. However, G\_ongoing takes the aspect of portability into account and hence, provides in addition an ongoing guarantee during the whole savings period.

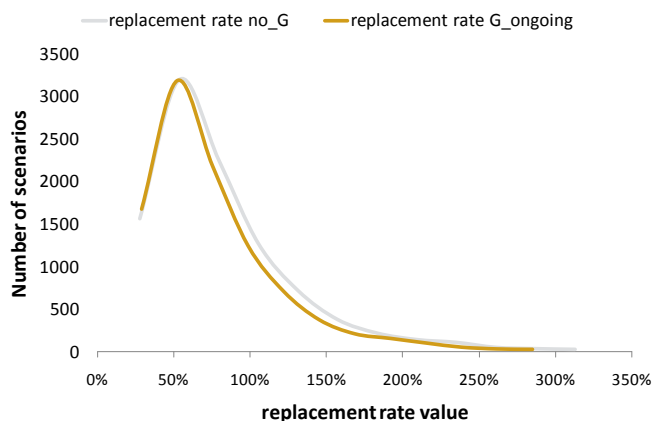


Figure 10: Empirical distribution of replacement rates G\_ongoing

	Replacement Rate		
	no_G	G_0	G_ongoing
	LC 80	LC 80	LC 80
1%-Q	22.65%	22.62%	24.54%
5%-Q	29.97%	29.66%	30.76%
25%-Q	47.72%	47.15%	46.11%
50%-Q	68.35%	67.49%	64.67%
75%-Q	100.39%	99.01%	94.24%
95%-Q	184.19%	181.51%	169.38%
99%-Q	287.82%	283.44%	263.15%
1%-CVaR	20.01%	20.72%	22.55%
5%-CVaR	25.56%	25.51%	26.99%
95%-CVaR	73.73%	72.77%	69.84%
99%-CVaR	79.64%	78.58%	75.21%
Mean	82.62%	81.52%	77.91%
STD	54.93%	54.04%	49.81%

Table 7: Statistical data for G\_0 and G\_ongoing

Although providing the same guarantees at maturity, the characteristics of the benefit payments for both types of guarantees vary significantly. G\_ongoing appears to be more conservative on the risk side but also allows less upside potential.



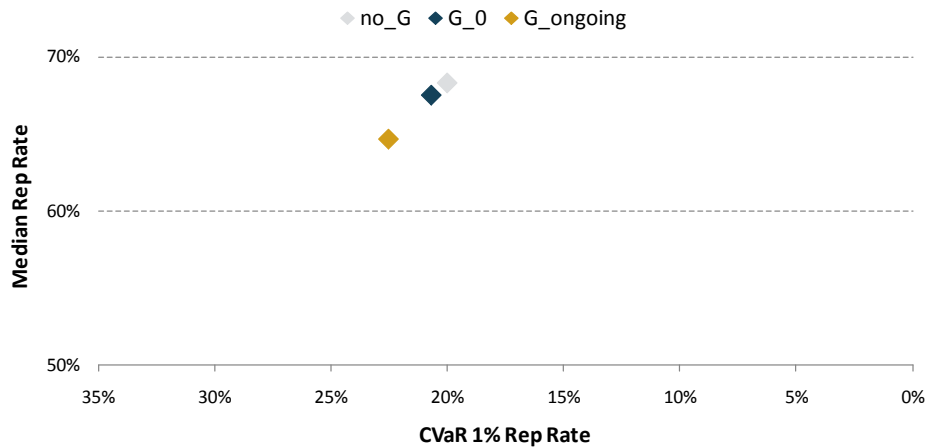


Figure 11: Risk return profile 1 for G\_0 and G\_ongoing

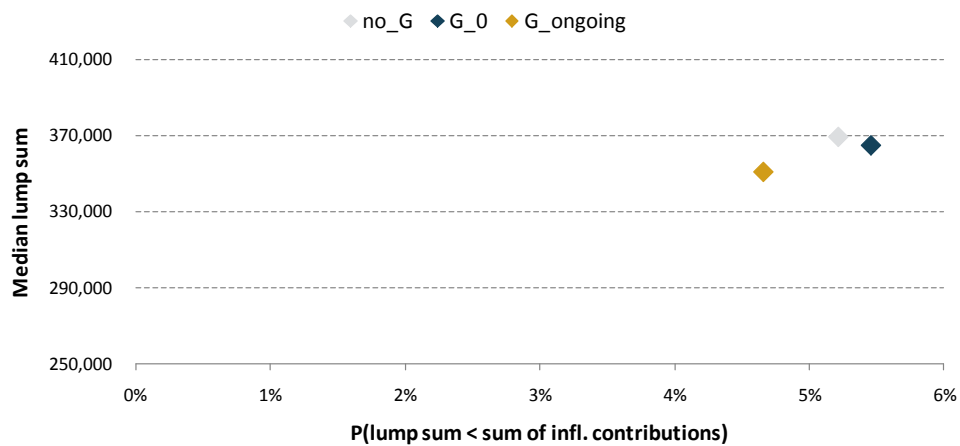


Figure 12: Risk return profile 2 for G\_0 and G\_ongoing

The location of G\_ongoing compared to G\_0 in both risk return profiles is similar as in both cases risk as well as return is reduced. From the data shown above we can derive that G\_ongoing is a very expensive guarantee from a plan member’s perspective. However, in this subsection we only focus on the benefits paid at retirement. In case the plan member leaves the plan (e.g. transferring the asset value to another plan), there is a risk that this value is relatively small – even in case the plan provides a guarantee. This risk is analyzed separately in Subsection 5.2.

#### 4.2.3 Low Level (G\_0) and High Level (G\_4)

In the following we analyze again G\_0 as it represents a very low guarantee level. Looking at G\_4 we have a rather high guarantee as the minimum return is defined to be 4% p.a.

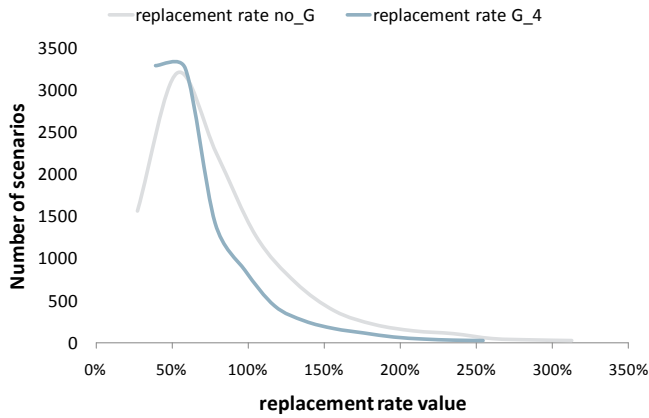


Figure 13: Empirical distribution of replacement rates G\_4

The benefit payments of G\_0 and G\_4 differ significantly. Regarding G\_4 low benefit payments are excluded due to the high guarantee level (cf. also very high values for VAR and CVAR for 1% and 5%). On the other hand the upside potential is very limited.

	Replacement Rate		
	no_G	G_0	G_4
	LC 80	LC 80	LC 80
1%-Q	22.65%	22.62%	35.35%
5%-Q	29.97%	29.66%	38.97%
25%-Q	47.72%	47.15%	46.57%
50%-Q	68.35%	67.49%	56.83%
75%-Q	100.39%	99.01%	80.77%
95%-Q	184.19%	181.51%	145.00%
99%-Q	287.82%	283.44%	224.23%
1%-CVaR	20.01%	20.72%	33.76%
5%-CVaR	25.56%	25.51%	36.73%
95%-CVaR	73.73%	72.77%	63.80%
99%-CVaR	79.64%	78.58%	68.19%
Mean	82.62%	81.52%	70.45%
STD	54.93%	54.04%	39.69%

Table 8: Statistical data for G\_0 and G\_4

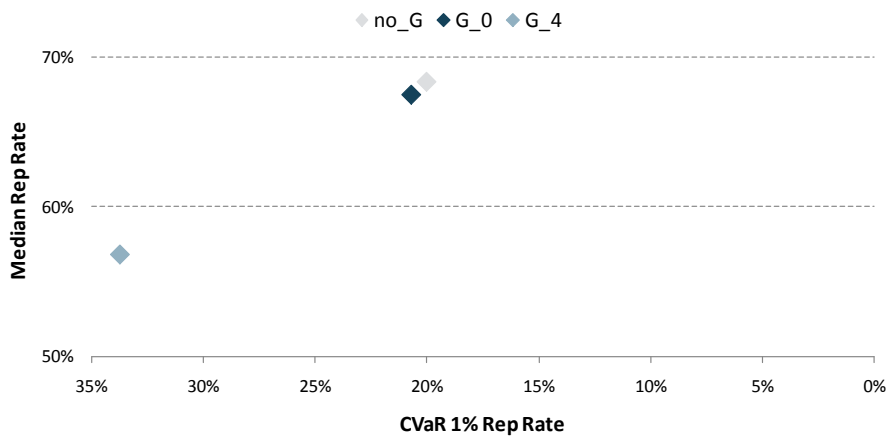


Figure 14: Risk return profile 1 for G\_0 and G\_4

The relatively high guarantee of G\_4 leads to significant lower return (in median). However, the risk of a lump sum below the sum of premiums in real terms is zero.

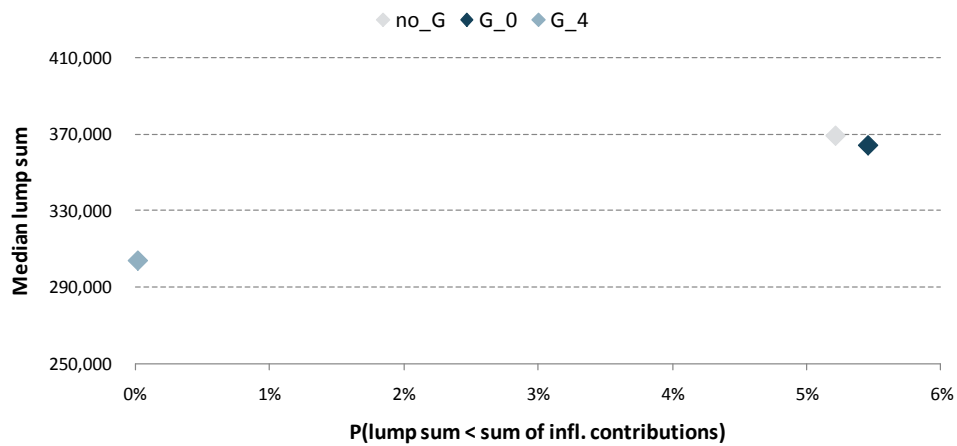


Figure 15: Risk return profile 2 for G\_0 and G\_4

Hence comparing a low level guarantee with a high level guarantee, the choice would depend on the risk aversion of a plan member and become a trade-off between risk and return as measured by the replacement rate.

#### 4.2.4 Fixed (G\_4) and Floating (G\_float)

G\_float defines the guarantee depending on the current capital market situation at the time of premium payment. The level of guaranteed benefits is similar for G\_4 and G\_float as we have seen it when comparing the mean in Subsection 3.4.

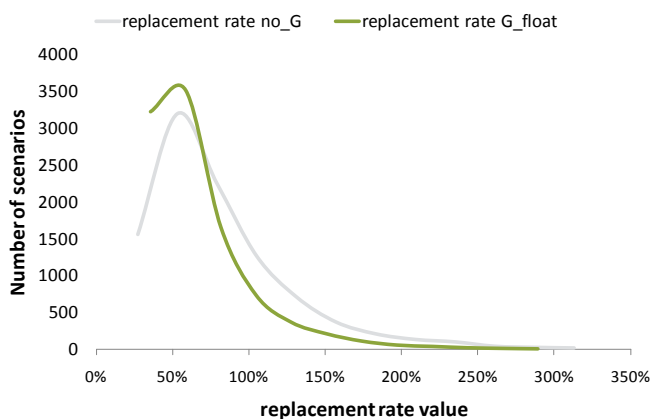


Figure 16: Empirical distribution of replacement rates G\_float

	Replacement Rate		
	no_G	G_4	G_float
	LC 80	LC 80	LC 80
1%-Q	22.65%	35.35%	29.83%
5%-Q	29.97%	38.97%	33.54%
25%-Q	47.72%	46.57%	43.68%
50%-Q	68.35%	56.83%	56.62%
75%-Q	100.39%	80.77%	79.60%
95%-Q	184.19%	145.00%	140.25%
99%-Q	287.82%	224.23%	218.27%
1%-CVaR	20.01%	33.76%	28.27%
5%-CVaR	25.56%	36.73%	31.16%
95%-CVaR	73.73%	63.80%	61.56%
99%-CVaR	79.64%	68.19%	65.80%
Mean	82.62%	70.45%	68.12%
STD	54.93%	39.69%	40.11%

Table 9: Statistical data for G\_4 and G\_float

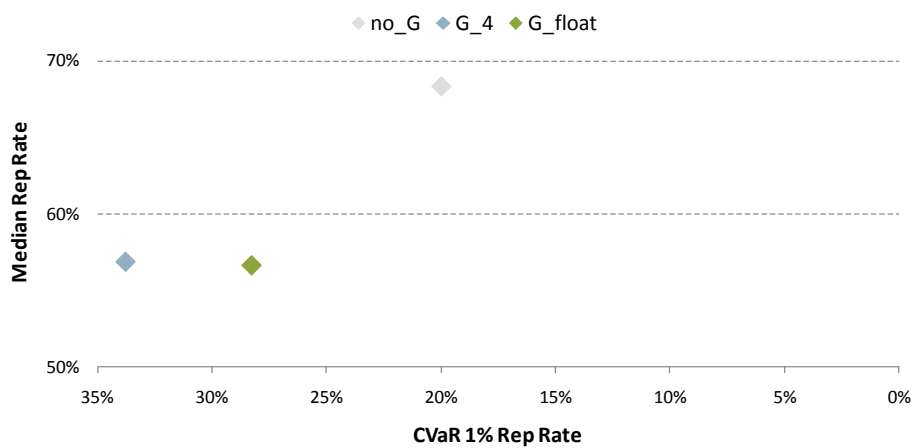


Figure 17: Risk return profile 1 for G\_4 and G\_float

Analyzing the statistical data risk return profiles, G\_4 appears to dominate G\_float (cf. the table and figure above) as the return is similar and the risk (measured as CVAR with a 99% confidence level) is higher for G\_float. In this setting these results can be explained by the guarantee fees, which are higher for G\_float compared to G\_4, although both of them provide similar guaranteed benefits in average (slightly higher for G\_float).

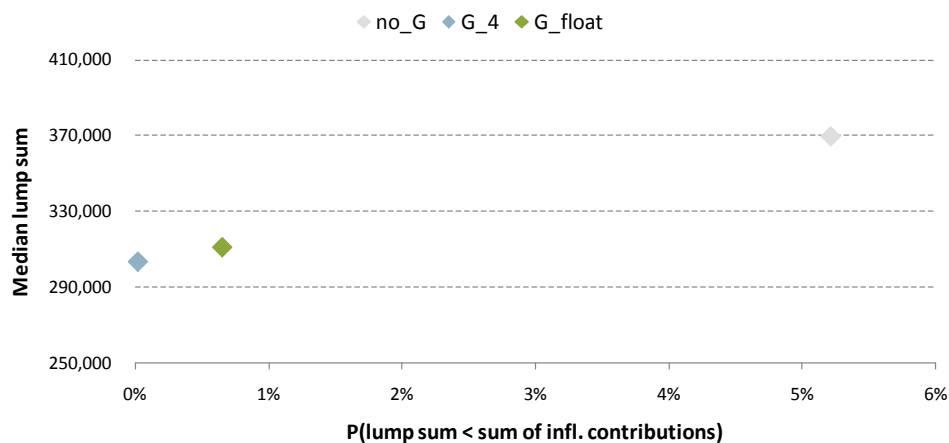


Figure 18: Risk return profile 2 for G\_4 and G\_float

As with G\_ongoing, G\_float has some characteristics which are not reflected in this analysis. To assess the value of a guaranteed interest rate depending on the current capital market development, the impact of different capital market scenarios has to be taken into consideration. Therefore we refer to Subsection 5.1 for a deeper understanding of the differences between G\_4 and G\_float, where specific scenarios are analyzed.

### 4.3 Sensitivity Analysis of Retirement Income regarding Model Parameters

The results shown in the previous subsections provide a first assessment on the basis of our standard modelling assumptions. Further analyses are necessary to underline the impact of our model setting and certain market risk factors on retirement income effects for a DC plan with different guarantees.

The next subsection deals with the effect of an investment horizon change, i.e. we reduce the savings phase period from 40 to 20 years. In Subsection 4.3.2, we show results for different underlying investment strategies.

#### 4.3.1 Effect on Retirement Income due to Changes of Investment Period

A change of length of the investment period has a significant impact on the guarantee fees and on the level of guaranteed benefits (as e.g. the sum of contributions changes). The following analysis addresses the effects on retirement income distributions using risk return diagrams. The risk is represented by the conditional value at risk at a confidence level of 99% of the replacement rate distribution and the return is

represented by the median value of the replacement rate distributions. The following figure summarizes the result for the different types of guarantees:

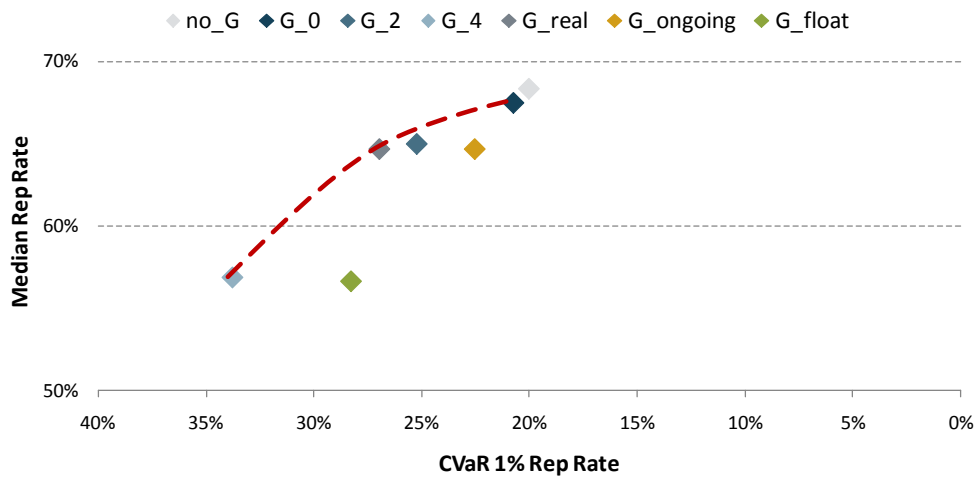


Figure 19: Risk return profile 1 for a savings phase of 40 years

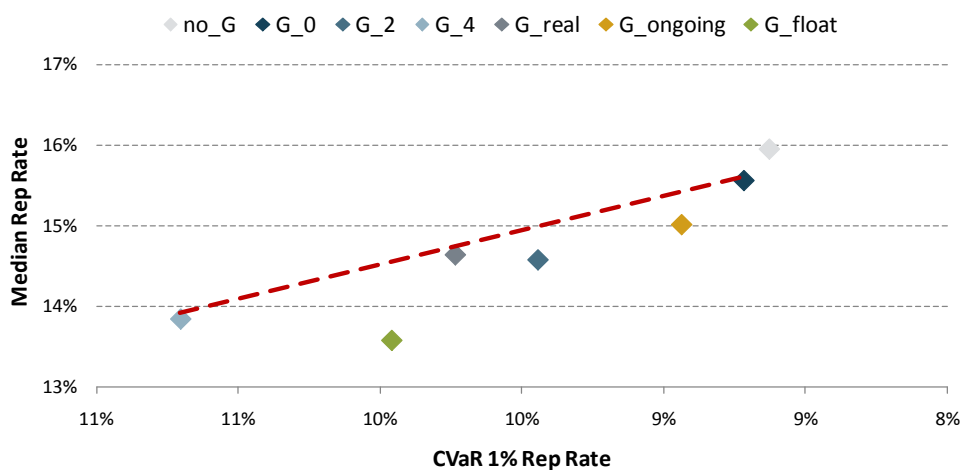


Figure 20: Risk return profile 1 for a savings phase of 20 years

In general, the level of risk increases and the level of return decreases due to the shorter investment horizon. Similar risk return profiles are observed when reducing the period of the savings phase. Given our setting, G\_float, G\_2 and G\_ongoing look inefficient in both situations when measured by return per units of risk.

When changing the risk measure, the conclusion drawn from the diagrams is changing. In the following figures, the risk is defined by the probability that the lump sum is below the sum of the contributions in real terms.

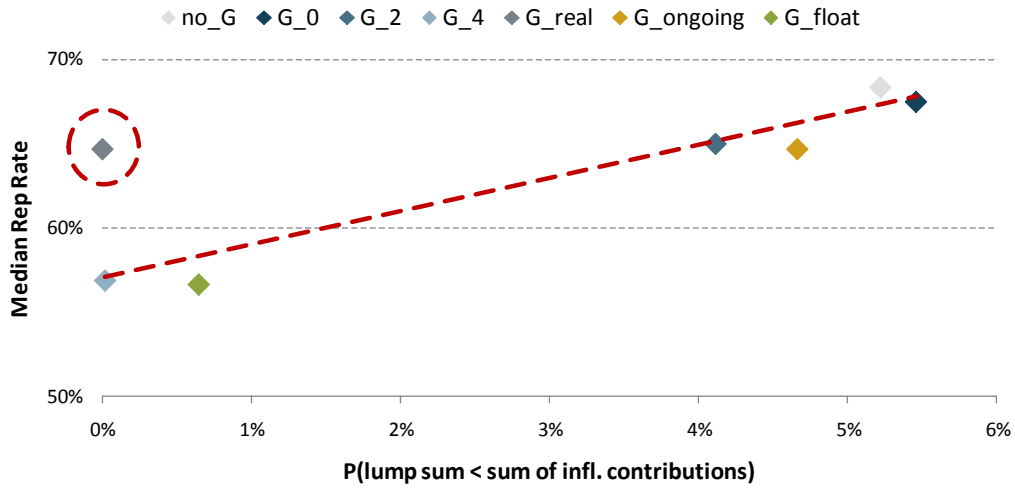


Figure 21: Risk return profile 2 for a savings phase of 40 years

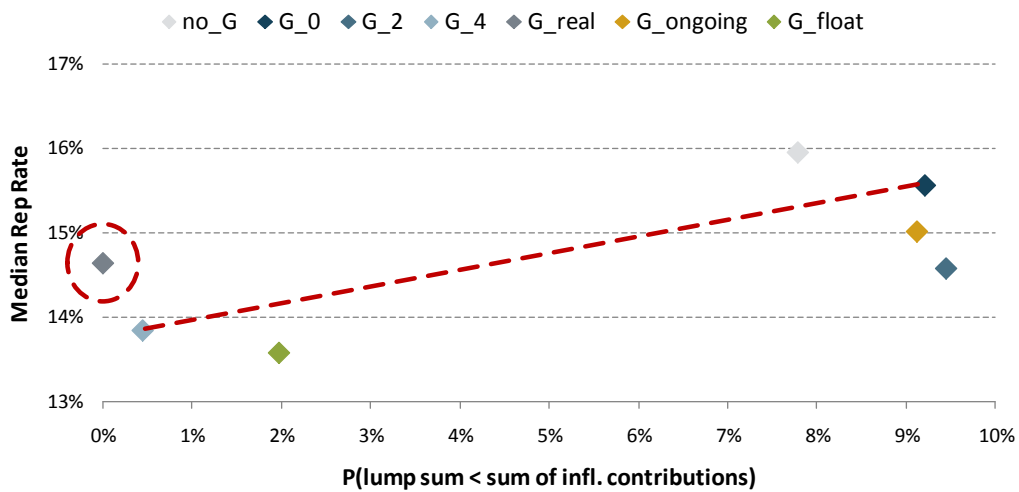


Figure 22: Risk return profile 2 for a savings phase of 20 years

Again, the level of risk increases and the level of return decreases due to the shorter investment horizon. Regardless of the investment period, G\_real appears to have an attractive risk return profile, which is due to the inflation protection characteristic of the guarantee (0% shortfall probability). Given our setting, the guarantees G\_float and G\_ongoing are inefficient in both situations. In a short investment period, G\_2 has the highest shortfall risk and is no longer efficient.

The statistical characteristics of the empirical replacement rate distributions for an investment horizon of 20 years are given in Appendix 7.3.

### 4.3.2 Effect on Retirement Income due to Changes of Investment Strategy

Corresponding to the effects of the guarantee fees, the change in type of investment strategy will alter the retirement income distribution. The effects will be illustrated using the familiar risk return diagrams.

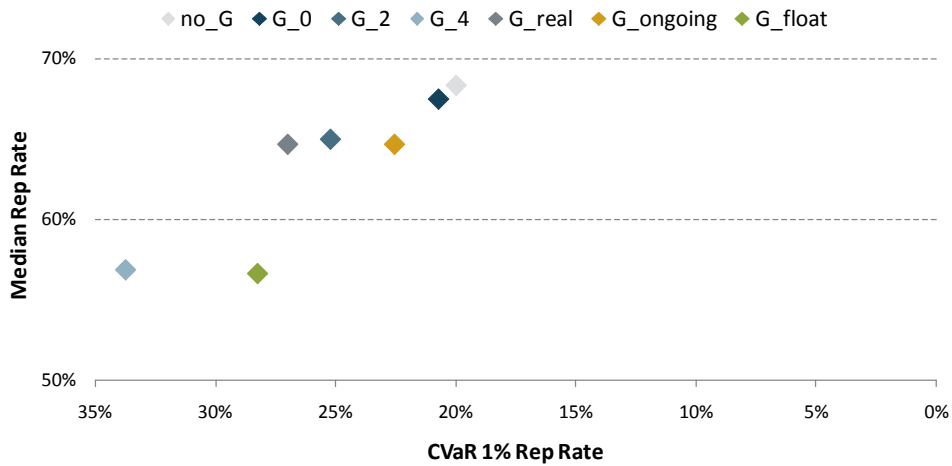


Figure 23: Risk return profile 1 associated with the LC 80 investment strategy

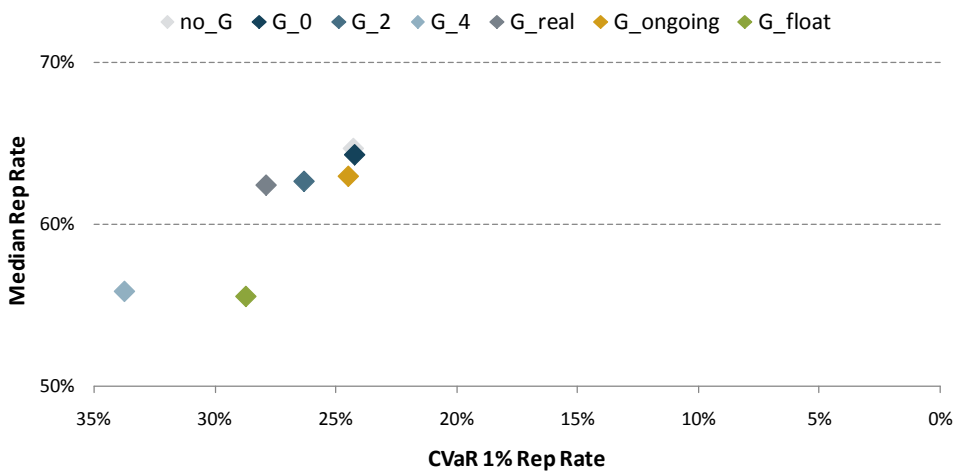


Figure 24: Risk return profile 1 associated with the LC 50 investment strategy



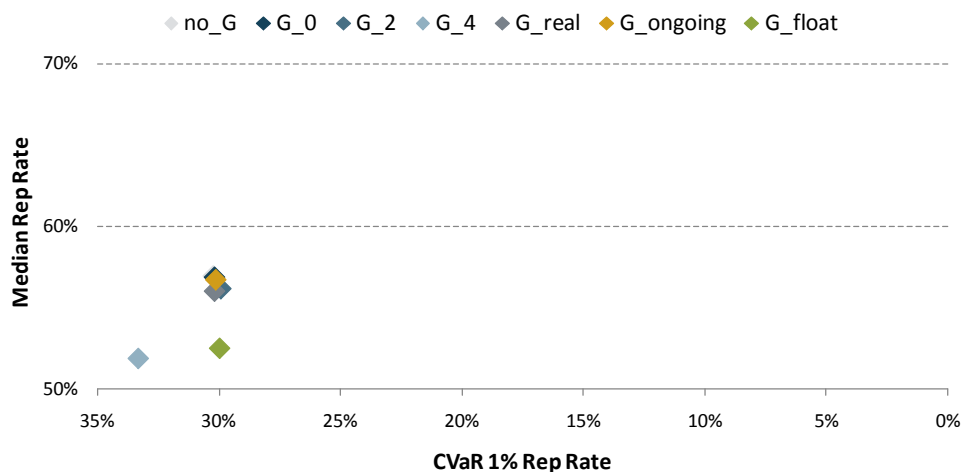


Figure 25: Risk return profile 1 associated with the LC 20 investment strategy

The median replacement rate value decreases as investments in bonds increases. As a result of a high investment in bonds, the replacement rate distribution becomes less volatile. G<sub>4</sub> provides a low downside risk although associated with a high cost as shown by the median replacement rate value. For LC 20, the risk is nearly the same for all types of guarantees apart from G<sub>4</sub>. For all strategies apart from G<sub>4</sub> and G<sub>float</sub> also the return is the same. G<sub>float</sub> appears to be inefficient in this setting.

The statistical characteristics of the empirical replacement rate distributions for both LC 50 and LC20 are given in Appendix 7.3.

## 5 Assessments of Retirement Income under Extended Perspectives

In this section, we continue our assessment of the retirement income results by extending the perspective. The retirement income distributions shown above looked quite similar for different types of guarantees even though the results might have been driven by very different economic scenarios. Since guarantees are purchased to protect against specific situations (like a hyper-inflation or to hedge against an equity crash), we have to assess the guarantee effect pathwise under such specific economic scenarios.

In the first subsection, we will focus on the single market (stress) scenarios to show the impact of our setting. In Subsection 5.2, we analyze the guarantees taken the portability aspect into account. A different design of charging the guarantee fees is investigated in Subsection 5.3, where we also compare guarantees by their accumulated guaranteed fees.

## 5.1 Retirement Income under Single Market Stress Scenarios

In addition to the results shown in Section 4, further analyses are necessary to underline the impact of our setting and certain risk factors. Hence we focus on the effect on retirement income under different market stress scenarios (see Subsections 5.1.2, 5.1.3, 5.1.4 and 5.1.5), which all have a declining equity return in common.

### 5.1.1 Expected Scenario

Considering expected capital market conditions, Figure 26 shows the outcome of this scenario in terms of replacement rate. It also states the guaranteed benefits by showing the corresponding lump sum.

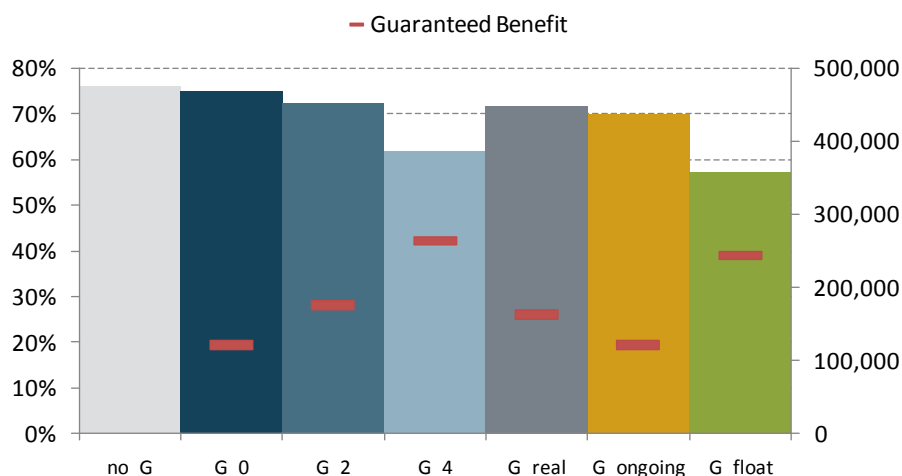


Figure 26: Benefits under expected capital market parameters

None of the replacement rates falls below the guaranteed level. The plan without guarantee (no\_G) provides the highest replacement rate, due to the fact that no guarantee fees are deducted. G\_real provides a similar replacement compared to G\_2, due to an inflation rate of about 2%. G\_float has a slightly lower replacement rate compared to G\_4, due to the shape of the nominal rate, which is convex over time in this scenario.

### 5.1.2 Scenario a: High Real Rate and Low Inflation Rate

In this first stress scenario real rate term structure keeps increasing and the inflation level is low compared to the expected scenario.

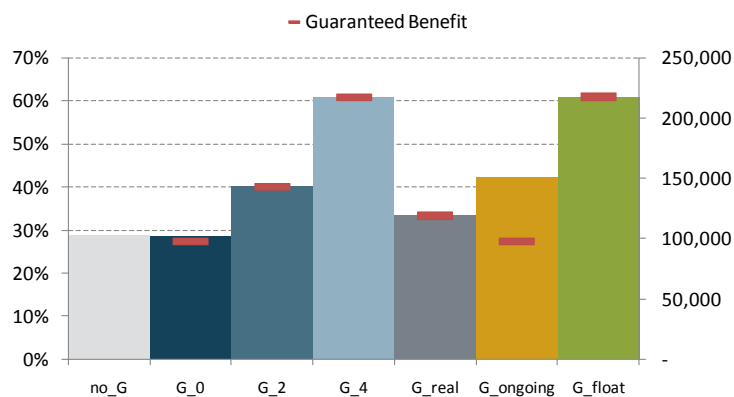


Figure 27: Benefits in a scenario with a high real rate and a low inflation rate

In this scenario for nearly all guarantee types, the benefits are determined by their guaranteed level. G\_float and G\_4 provide the highest replacement rate value due to their high guaranteed level. Their replacement rate values are similar because of the nominal rate. It is almost at the same level for both guarantees (i.e. about 4% p.a.), due to the high real rate and the low inflation rate. G\_real performs poorly compared to G\_2 due to the low inflation level. G\_ongoing provides a higher replacement rate than G\_0 due to its ongoing guarantees during the savings phase.

### 5.1.3 Scenario b: Low Real Rate and Inflation Rate Similar to the Expected Inflation

In this scenario, the real rate term structure keeps decreasing and the inflation level is similar to the expected inflation rate.

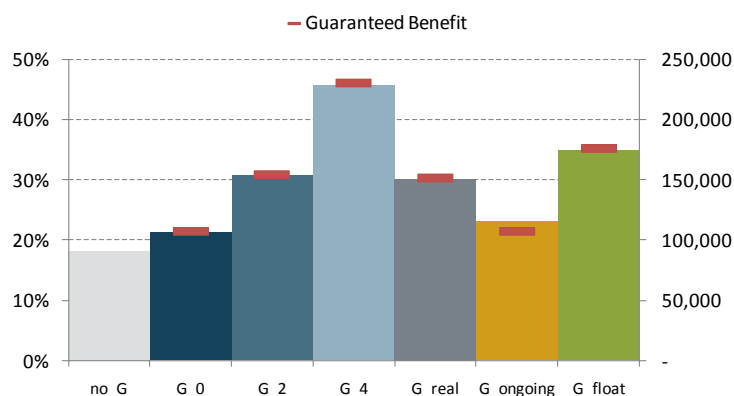


Figure 28: Benefits in a scenario with a low real rate and an inflation rate similar to the expected inflation

Again for nearly all guarantee types, the benefits are determined by their guaranteed level. G\_4 provides the highest replacement rate value due to their high guaranteed

level. G\_float performs poorly compared to G\_4 as the 1-year real rate decreases over time. G\_real and G\_2 have similar replacement rate values, as they provide similar guaranteed levels. In this scenario G\_0 is preferable to no\_G as the guarantee enables higher benefits.

#### 5.1.4 Scenario c: Low Real Rate and High Inflation Rate

In this market stress scenario, the real rate term structure keeps decreasing and the inflation level is relative high compared to the expected scenario.

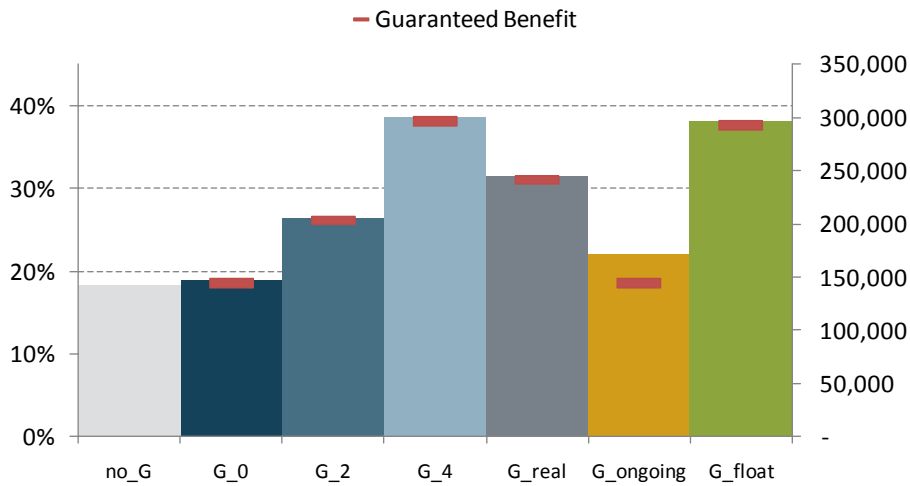


Figure 29: Benefits in a scenario with a low real rate and a high inflation rate

In this stress scenario, again for nearly all guarantee types, the benefits are determined by their guaranteed level. G\_float and G\_4 provide the highest replacement rate value due to their high guaranteed level. Their replacement rate values are similar because of the nominal rate. It is almost at the same level for both guarantees (i.e. about 4% p.a.), due to the low real rate and the high inflation rate. As expected, G\_real provides an inflation protection. Thus the replacement rate value of G\_real is higher than the one of G\_2. G\_ongoing provides a higher replacement rate than G\_0 due to its ongoing guarantees during the savings phase.

#### 5.1.5 Scenario d: High Real Rate and Inflation Rate Similar to the Expected Inflation

In our last stress scenario the real rate term structure is relatively high and the inflation level is similar to the expected inflation rate.

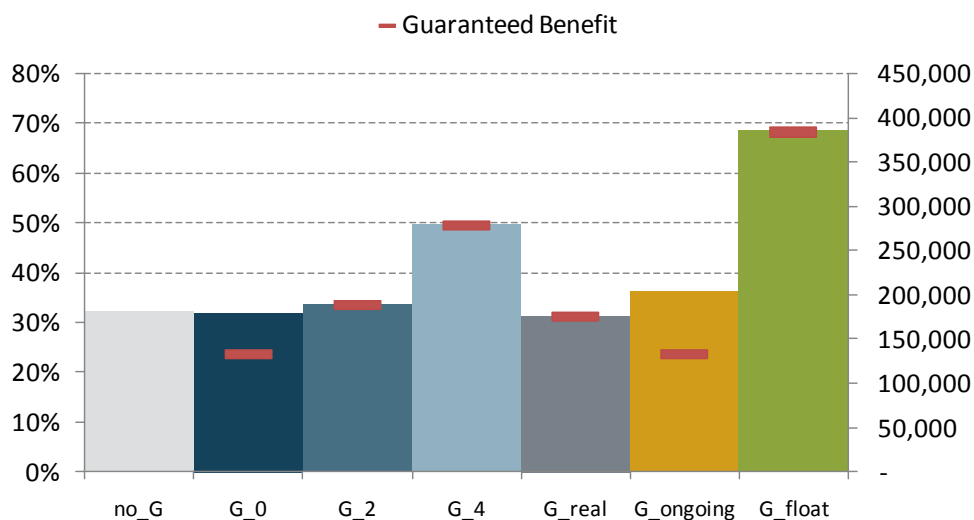


Figure 30: Benefits in a scenario with a high real rate and an inflation rate similar to the expected inflation

G\_float has a significantly higher replacement rate value compared G\_4, due to the higher nominal rate of G\_float compared to 4% p.a. Indeed in case of high real rates combined with an expected inflation rate of 2%, the nominal rate tends to be higher than 4%. G\_real has a similar replacement rate value compared to G\_2. Though the performance of G\_real is slightly lower due to the exponentially form of the inflation rate curve. G\_ongoing provides a higher replacement rate than G\_0 due to its ongoing guarantees during the savings phase.

## 5.2 Retirement Income under Portability Options

So far, we have only analyzed the benefits at the beginning of retirement. However, it might occur that a plan member leaves the plan transferring his assets to another pension fund. In this case, the plan member is exposed to the risk of a low value of assets, e.g. smaller than the amount of contributions paid in so far. Hence, in the following we analyze the probability that the net asset value (NAV) is below the sum of contributions paid after 5 years, hence assuming a transfer 5 years after commencement of the plan.

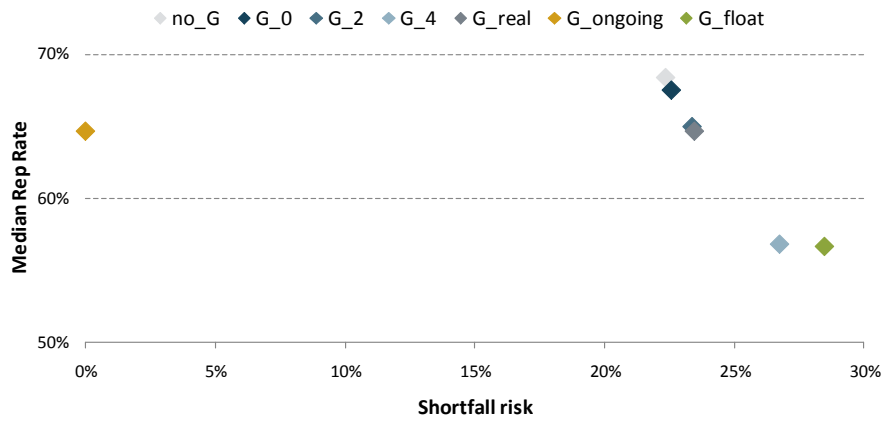


Figure 31: Risk return profile 1 regarding portability aspects

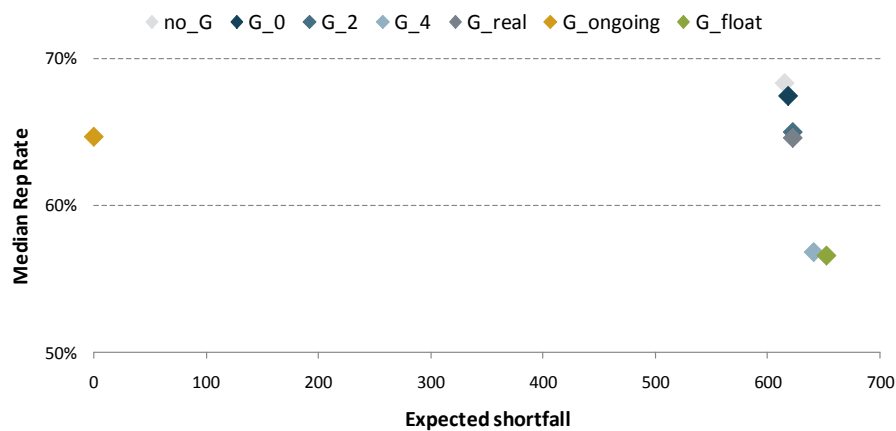


Figure 32: Risk return profile 2 regarding portability aspects

There is no shortfall risk for G\_ongoing due to its design. It also provides a good replacement rate (in median) compared to the other guarantees. The expected shortfall for all guarantees (except G\_ongoing) is very similar. From a portability perspective one would focus on the guaranteed level during the entire savings phase. Under this view G\_ongoing shows clear advantages.

Given this conclusion, a deeper focus on the design of the guarantee G\_ongoing is needed, especially concerning its special feature to guarantee a 0% nominal rate on contributions in each year during the savings phase. If G\_ongoing looks attractive when looking at the portability aspect, it would be interesting to know the probability that the guarantee is activated at least one time during the whole savings phase (measured by numbers of scenarios where this event occurs). The following table summarizes those figures. Hence, there is for instance 83.5% risk that the guarantee is activated at least once for the 40 years savings period with a LC 80 investment strategy (i.e. in about

8'350 of 10'000 scenarios). The more conservative the investment strategy gets, the lower the probability gets.

	Investment Horizon 40Y	Investment Horizon 20Y
LC 80	83.5%	82.9%
LC 50	66.3%	xx
LC 20	24.5%	xx

Table 10: Probability that the guarantee G\_ongoing is activated at least once

The probability that the guarantee is activated at least 4 (resp. 7) times within a scenario during the whole savings phase is summarized in the Table 11 (resp. Table 12).

	Investment Horizon 40Y	Investment Horizon 20Y
LC 80	30.0%	24.0%
LC 50	8.9%	xx
LC 20	0.1%	xx

Table 11: Probability that the guarantee G\_ongoing is activated at least 4 times

	Investment Horizon 40Y	Investment Horizon 20Y
LC 80	7.5%	3.6%
LC 50	0.8%	xx
LC 20	0.0%	xx

Table 12: Probability that the guarantee G\_ongoing is activated at least 7 times

The maximum, the median and the minimum number of years that the guarantee is activated during the savings phase is summarized in the Table 13.

	Investment Horizon 40Y	Investment Horizon 20Y
LC 80	17/2/0	13/2/0
LC 50	13/1/0	xx
LC 20	4/0/0	xx

Table 13: Maximum/Median/Minimum number of years, where the guarantee G\_ongoing is activated during the savings phase

The guarantee is activated in median in only few years (0, 1 or 2). However, these figures show in the worst case a large number of years of activation, which decreases as the underlying investment strategy gets more conservative or when the savings period decreases.

### 5.3 Effects of Haircut Fee Schedule on Retirement Income

Typically, when purchasing a guarantee the buyer has to pay an explicit guarantee fee based either on NAV or on contributions paid. However, we observe that some providers of DC plans with guarantees (e.g. German or Swiss pension funds (Pensionskassen)) do not charge an explicit fee but instead get compensated by participating with a pre-defined fraction of the excess return.

In this subsection, we compare two approaches based on G\_4 as presented above, i.e. the required guarantee is a minimum return guarantee of 4% p.a. As an alternative financial design of paying for the guarantee the following so called haircut financing strategies are investigated:

- Final Haircut: The guarantee provider receives an amount of the surplus at the beginning of retirement, where the surplus is the difference of NAV (i.e. lump sum) and guaranteed benefits.<sup>19</sup>
- Ongoing Haircut: The guarantee provider receives an amount of each year's surplus, where the surplus is the difference of the current NAV and the guaranteed benefits (based on contributions paid).<sup>20</sup>

	Final Haircut	Ongoing Haircut
Investment Horizon 40Y, LC 80	24.06%	1.60%
Investment Horizon 40Y, LC 50	22.02%	1.63%
Investment Horizon 40Y, LC 20	18.87%	1.57%
Investment Horizon 20Y, LC 80	83.27%	18.95%

Table 14: Guarantee fees for G4 based on haircut financing design

Note: considering the base case (IH 40Y, LC80), a parallel shift of +10% in the volatility term structure would increase the guarantee fees by almost 17% and 10% resp. for the final haircut and the ongoing haircut strategies. A parallel shift of -1% in the interest rate term structure would increase the guarantee fees by almost 89% and 136% resp. for the final haircut and the ongoing haircut strategies. A parallel shift of +1% in the inflation term structure would increase the guarantee fees by almost 1% and 4% resp. for the final haircut and the ongoing haircut strategies.

From the table above we can say that, in case of LC 80 and an investment horizon of 40 years, about 24% of the final surplus (if there is any) is deducted in order to finance a guarantee of 4% p.a. For a shorter time horizon, the volatility risk has more impact than

<sup>19</sup> Note that the results in the following are on a theoretical basis only as in reality such financing of guarantees cannot be found.

<sup>20</sup> Note that with this surplus definition the present value of guaranteed benefits is not used. As a consequence the surplus is very low in the first years. In the German or Swiss system mentioned above the definition of the surplus differs significantly.



for a 40 years investment period, since the drift has more impact than the volatility risk for the longer period. The main effect will be that guarantees fees will increase.

The following figure illustrates the replacement rate distributions of G\_4, G\_final\_haircut and G\_ongoing\_haircut for an investment horizon of 40 years with the investment strategy LC 80.

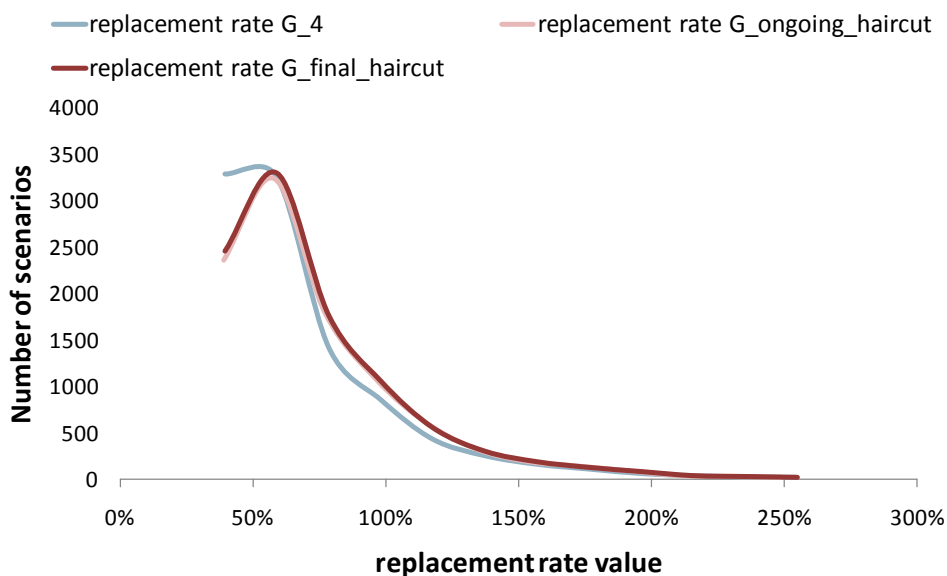


Figure 33: Empirical distributions of replacement rates for G\_4 and both haircut strategies

The guarantee level for G\_4 and the haircut guarantees are the same. However, from the empirical distribution of replacement rates as well as from the statistical data shown below, we can say that both haircut strategies are not only very similar but also dominate G\_4.

	Replacement Rate		
	G 4	G o hc	G f hc
	LC 80	LC 80	LC 80
1%-Q	35.35%	35.96%	36.01%
5%-Q	38.97%	40.09%	40.27%
25%-Q	46.57%	49.31%	49.51%
50%-Q	56.83%	63.20%	63.24%
75%-Q	80.77%	88.19%	88.03%
95%-Q	145.00%	150.63%	152.41%
99%-Q	224.23%	223.22%	232.66%
1%-CVaR	33.76%	34.42%	34.50%
5%-CVaR	36.73%	37.57%	37.68%
95%-CVaR	63.80%	68.83%	68.87%
99%-CVaR	68.19%	73.20%	73.36%
Mean	70.45%	75.38%	75.63%
STD	39.69%	40.18%	41.11%

Table 15: Statistical data for G\_4 and G\_float

Note, the advantages being observed of haircut strategies can be reasoned as follows. The difference of the benefits is not due to the haircut financing (i.e. surplus as basis of the fee) but because of the different payment schemes for the guarantee costs. For both haircut strategies the guarantee fees are deducted mostly during the end of the savings phase (in case of final haircut at the time of retirement only). That means that in both cases, the NAV is mostly higher compared to G\_4's NAV and, hence, the haircut strategies profit from the return above the risk free rate. However, the haircut financed guarantees do not only appear to be much better because of modeling reasons but also because issues regarding solvency for instance are not included in this analysis. Indeed in reality such extreme back loaded financing would not happen due to various reasons, such as need of equity capital burden required by solvency rules during the whole period, sales incentives (which have to be paid upfront), counterparty risk against the plan member's behavior regarding pre-mature cancelation, etc.

In order to compare the cost of guarantees in case the basis of the fees differs (NAV vs. surplus) an aggregated cost measure has been calculated. This measure is based on the amount of guarantee fees paid in currency units each year.

For each scenario, the sum of the discounted guarantee fees is computed and divided by the lump sum where no guarantee option is being chosen:

$$\text{Guarantee Cost}_s = \frac{1}{\text{Lump Sum}_s} \sum_{t=25}^{65} (\text{Guarantee Fees}_{s,t}) \times \text{Inflation Index}_{s,65,t}, \forall s \in \{1, \dots, 10000\}$$

Hence, we get the sum of relative guarantee costs for each real world scenario. The following table illustrates statistical data of guarantee fees paid (in real terms) in relation to the lump sum. All figures are calculated for our base case (i.e. for the investment strategy LC80 and a savings phase of 40 years).

	Cost of Guarantee								
	no_G LC 80	G_0 LC 80	G_2 LC 80	G_4 LC 80	G_real LC 80	G_ongoing LC 80	G_float LC 80	G_ongoing_haircut LC 80	G_final_haircut LC 80
Mean	0.00%	0.89%	3.43%	12.55%	3.78%	6.26%	16.45%	5.93%	7.68%
1%-Q	0.00%	0.56%	2.16%	7.79%	2.38%	3.88%	10.11%	0.02%	0.00%
5%-Q	0.00%	0.63%	2.44%	8.82%	2.69%	4.37%	11.49%	0.23%	0.00%
25%-Q	0.00%	0.76%	2.92%	10.63%	3.21%	5.28%	13.89%	2.54%	1.37%
50%-Q	0.00%	0.86%	3.33%	12.20%	3.67%	6.08%	15.96%	5.74%	7.67%
75%-Q	0.00%	0.99%	3.84%	14.10%	4.23%	7.03%	18.52%	8.85%	12.65%
95%-Q	0.00%	1.23%	4.75%	17.52%	5.24%	8.81%	23.12%	12.57%	17.47%
99%-Q	0.00%	1.43%	5.54%	20.52%	6.11%	10.27%	27.04%	15.39%	19.83%
1%-CVaR	0.00%	0.53%	2.05%	7.35%	2.25%	3.65%	9.52%	0.01%	0.00%
5%-CVaR	0.00%	0.59%	2.27%	8.19%	2.50%	4.07%	10.64%	0.10%	0.00%
95%-CVaR	0.00%	0.86%	3.33%	12.19%	3.67%	6.08%	15.97%	5.49%	7.09%
99%-CVaR	0.00%	0.88%	3.40%	12.46%	3.75%	6.21%	16.32%	5.81%	7.55%

Table 16: Comparison of aggregated guarantee costs

The aggregated guarantee costs for all guarantees, where fees are based on the NAV are consistent to the results shown in the previous Subsection 3.3. More interesting are the haircut strategies compared to G\_4 as they all provide the same level of guarantee. In case of a final haircut no guarantee fee is deducted in at least 5% of the scenarios. Both haircut strategies need less guarantee fees compared to G\_4, which underlines our results above where the haircut strategies had preferable empirical distribution of replacement rates in the real world simulation. However, also regarding this result we have to point out, that this is an effect which is due to our model design not including all operational cost components of such a different fee concept. In addition, we have to emphasize that the results shown above are very sensitive to the discount factor that is used in the formulae.

## 6 Conclusions and Discussion

In order to assess the effects of guarantee components on retirement income in DC pension plans, we analyzed a set of six structurally different types of guarantees. The various perspectives taken in this analysis underline the difficulty to provide a unique and comprehensive assessment that enables to derive a ranking of the cost-benefit effects of the different guarantees. As guarantees are typically purchased in order to hedge against a specific real world risk (or unfavourable future economic scenario), assessments need to be based on the respective perspectives.

When comparing different types of guarantees one can either look from the view of the guarantee provider (i.e. the pricing) or from the view of the plan member. In this study, we considered both sides. Using the theory of fair pricing of guarantees in a risk-neutral framework, we calculated the necessary guarantee fees for a given guarantee concept. This allowed us to express a price of a guarantee as a percentage of the net asset value (NAV). Thus, we can identify cheap guarantees with low guarantee fees, such as G\_0 (which provides only an asset value guarantee). In the base setting, G\_4 (minimum return of 4% p.a.) and G\_ongoing (asset value guarantee during the whole savings phase) for example, belong to the most expensive types of guarantee.

In addition, we set a Monte Carlo simulation framework based on 10'000 real world scenarios in order to illustrate the benefit payments for a given guarantee type.

In the context of funded private pension and retirement systems, replacement rates are a measure commonly being used. Therefore we analyzed the possible replacement rates, i.e. their resulting empirical distribution and the corresponding statistical values, such as value at risk or conditional value at risk. In order to illustrate the return as well as

the risk simultaneously, we used risk return profiles to compare the different types of guarantees. The analyzed guarantees were grouped given their main characteristics:

- Nominal (G\_0) and Real (G\_real)
- At Retirement only (G\_0) and Ongoing during the Savings Phase (G\_ongoing)
- Low Level (G\_0) and High Level (G\_4)
- Fixed (G\_4) and Floating (G\_float)

As a result, we can recognize guarantees with low risk and low return (such as G\_4), guarantees with high risk and also high return (such as G\_0) and of course, guarantees in between. Some guarantees like G\_float and G\_ongoing appear to be inefficient within this analysis. However, characteristics of guarantees which depend on the development of the capital market (as G\_real, where the guarantee depends on inflation or G\_float, where the guarantee depends on the risk free rate) cannot be assessed sufficiently hereby.

The focus of a DC pension plan and any included guarantee is of course on the benefits paid at retirement. However, in reality, it might happen that a plan member leaves the plan transferring his assets to another pension fund. In this case, the plan member is exposed to the risk of a low value of assets. Adding this perspective to the assessment of guarantee, G\_ongoing becomes much more attractive as it exactly limits this risk.

The price of a guarantee is extremely sensitive to the current capital market. As we have shown, the guarantee fees change significantly depending on volatility, interest rates and inflation. However, such changes do not affect all types of guarantees to the same extent. For instance, a parallel shift of -1% of the interest rate term structure leads to a reduplication of guarantee fees for G\_0 but leaves the fees for G\_float basically unaffected.

This issue is important for the guarantee providers as well as for plan members. A significant change of important capital market parameters could, on the one hand, lead to a re-pricing, that could affect the benefit payments of the plan members. On the other hand, if guarantee fees would remain unchanged, the guarantee provider would bear the risk of insufficient guarantee fees.

Monte Carlo simulation is one method to illustrate possible benefit payments. Of course the underlying scenarios assume a certain capital market development, which is in median often an expected capital market assumptions. However, the suitability of a guarantee depends on the individual market expectations. As we have shown by analyzing selected scenarios, the guarantees behave differently due to their design.

The guarantee G\_float for instance, provides a guarantee level which depends on the current risk free rate at the time of contribution payment. In scenarios with a high real

rate and an expected level of inflation, the guarantee is relatively high (in our case also higher than the guarantee G\_4). On the other hand, in a scenario with a low real rate and an expected level of inflation, the provided guarantee is relatively low as the risk free rate is rather low.

When assessing guarantee fees or benefit payments, the guarantees G\_real and G\_2 (minimum return of 2% p.a.) often appear to be very similar as the inflation level is about 2% in our model. An analysis of specific scenarios allows us to highlight their different characteristics. In a scenario with low inflation rate, G\_2 performs better compared to G\_real, since the benefit payments are higher and vice versa. In a scenario with high inflation rate, G\_real provides high guarantees and benefits are higher than the guarantee G\_2.

The results shown in this study give a comprehensive assessment of different types of guarantees within DC pension plans. Different perspectives are necessary in order to capture the relevant issues. Further analyses could take other issues into account. For instance, the analyses could be extended by modelling also solvency capital requirements. The necessary capital depends significantly on the given guarantee. However, as these requirements depend on a specific country, we omitted this issue. Another extension could be the analysis of the longevity risk, which is an important factor both from the plan member's and the guarantee provider's view.

## 7 Appendix

### 7.1 Capital Market Models

In this study, we introduce two different models. A risk neutral model<sup>21</sup> is used to determine the fair price of the guarantees being investigated, whereas real world scenarios derived from the economic scenario generator<sup>22</sup> are used to bring a view on the economic environment in order to assess the guarantee benefits in terms of replacement rate.

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<sup>21</sup> The risk neutral model is described in Appendix 7.1.1.

<sup>22</sup> The economic scenario generator is described in Appendix 7.1.2. Characteristics of real world scenarios are given Appendix 7.1.3.

### 7.1.1 Risk Neutral Model<sup>23</sup>

To determine prices of the optional components of the guarantees, we need to specify a capital market model. Several risk factors are included and we use the following stochastic processes to model them:

- interest rates:  $dr(t) = (\theta(t) - ar(t))dt + \sigma_R dW_R(t)$
- equities:  $dS_i(t) = \mu_i(t)S_i(t)dt + \sigma_{S_i}(t)S_i(t)dW_{S_i}(t)$
- inflation:  $di(t) = (\theta_i(t) - a_i i(t))dt + \sigma_I dW_I(t)$

where the Brownian Motions ( $W_R$ ,  $W_{S_i}$ ,  $W_I$ ) are correlated. The model is based on the Hull-White model for the short-term interest rate and short term inflation and on an extended Black-Scholes model for the dynamics of equity returns. Using the Girsanov Theorem, it can again be shown that there exists a unique probability measure under which the processes satisfy the following stochastic differential equations:

- $dr(t) = (\theta(t) - ar(t))dt + \sigma_R dW_R(t)$
- $\frac{dS_i(t)}{S_i(t)} = r(t)dt + \sigma_{S_i}(t)dW_{S_i}(t)$
- $di(t) = (\theta_i(t) - a_i i(t))dt + \sigma_I dW_I(t)$

Note that the Brownian motions ( $W_R$ ,  $W_{S_i}$ ,  $W_I$ ) are correlated with the same correlation as they are under the real-world perspective. This ensures compatibility with the analysis of the retirement income results in Sections 4 and 5.

The assumptions of the risk-neutral model and the economic scenario generator introduced in the next subsection are basically the same. They use for example the same interest rate curve and inflation curve as inputs for the calibration.

The purpose of the risk-neutral model is an arbitrage-free pricing of the guarantees, whereas the economic scenario generator is used to adequately produce capital market scenarios including risk premiums for the different asset classes.

### 7.1.2 Economic Scenario Generator

Scenario analyses have become a powerful tool to assess complex financial decision situations. With increased computational power, intensive analyses with many economic variables over several time steps have meanwhile become a feasible task.

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<sup>23</sup> For an introduction to the probabilistic theory behind the risk-neutral valuation principles see for example: N. H. Bingham, R. Kiesel (2004), 'Risk-Neutral Valuation – Pricing and Hedging of Financial Derivatives'.

The quality of the results and conclusions drawn upon depend heavily on the economic scenarios assumed within the analysis. So the underlying concept of the scenario generation is crucial. Different Economic Scenario Generators (ESGs) have been developed and described in the literature.<sup>24</sup> Among the available ESGs, three classes can be distinguished in terms of the underlying model:

- The econometrics-based models.
- The pricing-based models.
- The hybrid models.

The first one is based on statistical and econometric theory.<sup>25</sup> The main advantage of this ESG class is its simplicity. However, a fully historical-based method might not be suitable or at least not robust enough for forecasting purposes because it depends on the number of available observations. Besides, this kind of models cannot be used to price financial instruments since simulations under the risk-neutral measure are not possible.

The second class brings a solution for the latter issues. Indeed, these models are based on stochastic mathematical tools which allow risk-neutral calculations. The pricing-based models<sup>26</sup>, unlike the econometrics-based ones, can accommodate for both expressing historical information as well as future expectation. However, risk factors, e.g. inflation or interest rate, may not be in line with global understanding of the market since the focus of these models is put on pricing.

Thus a third class of ESGs evolved. The hybrid model, where risklab ESG<sup>27</sup> is one example, tries to combine the main advantages of both previous classes. Indeed the model, which is implemented in the risklab ESG, aims to overcome the drawback of the pricing-based ESGs by including the influence of macroeconomic variables in the integrated modelling framework. It combines both statistical and financial theory in the sense that it uses observable financial variables like inflation to describe the evolution of other economic or pricing variables that are not necessarily observable. To achieve this kind of sophistication, the underlying processes are modelled by Stochastic differential

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<sup>24</sup> A.D. Wilkie, M.A., F.F.A., F.I.A. (1995), 'More on a stochastic asset model for actuarial use', *British Actuarial Journal*, 1, 777-964.

J. Hibbert, P. Mowbray, C. Turnbull (2001), 'A stochastic asset model & calibration for long-term financial planning purposes'.

R. Zagst, S. Antes, B. Schmid, M. Ilg (2008), 'Empirical evaluation of hybrid defaultable bond pricing models', *Journal of Applied Mathematical Finance*.

<sup>25</sup> The Wilkie Investment Model, very popular in the United Kingdom, is a prominent representative of these kinds of models. The first version was released in 1986 and the second in 1995.

<sup>26</sup> The Barrie and Hibbert Model is a prominent representative of these kinds of models. Insights have been published in 2001.

<sup>27</sup> The risklab economic scenario generator is a proprietary ESG which is enhanced in cooperation with Prof. Dr. Rudi Zagst, Professor of Mathematical Finance at the Munich University of Technology.

equations (SDE) with parameters that are estimated gradually following a cascade structure as shown in Figure 34.

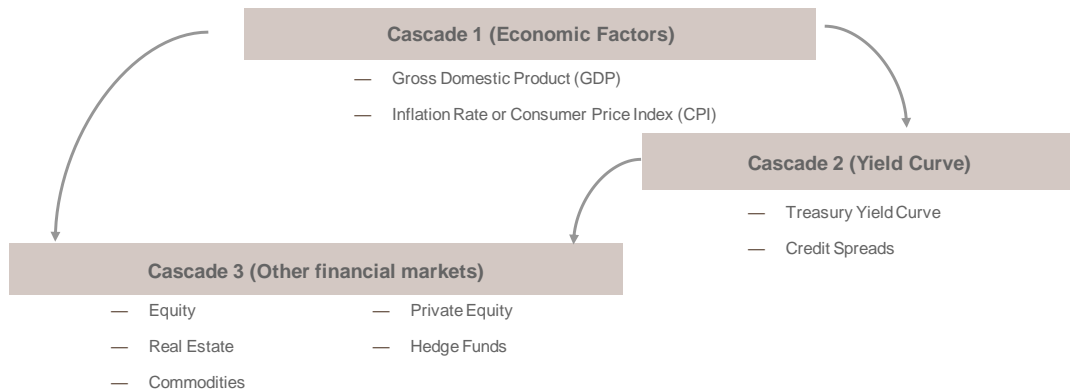


Figure 34: Cascade structure used to model the risklab ESG

The advantage of such integrated models is to allow the estimation of a complex market model with different interdependent factors step by step. Thus if inflation is a top generating economic factor, then all the factors modelled below it might depend on inflation. All the processes in Cascade 1 and Cascade 2 are modeled with a mean reversion property in mind which means that the processes tend towards their long-term mean. Such behaviour can e.g. be observed in the market for interest rates or the GDP.

Cascade 1 deals with the macroeconomic parameters such as the Gross Domestic Product (GDP) and the inflation. The dynamics of the GDP growth rate  $r_\omega$  and the dynamics of the inflation rate  $r_i$  are respectively given by the following Vasicek model<sup>28</sup>:

$$dr_\omega(t) = (\theta_\omega - a_\omega r_\omega(t))dt + \sigma_\omega dW_\omega(t) \quad (1)$$

$$dr_i(t) = (\theta_i - a_i r_i(t))dt + \sigma_i dW_i(t) \quad (2)$$

where  $W_\omega = (W_\omega(t))_{t \geq 0}$  and  $W_i = (W_i(t))_{t \geq 0}$  are Wiener processes. The mean reversion

levels are given by  $\frac{\theta_\omega}{a_\omega}$  for (1) and  $\frac{\theta_i}{a_i}$  for (2).

Cascade 2 deals with the treasury yield curves and the credit spreads. Concerning the treasury yield curve the real short rate  $r_R$  dynamics is given by a two-factor Hull-White model<sup>29</sup>.

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<sup>28</sup> The Vasicek model describes the evolution of rates and was the first one to capture the mean reversion characteristic. It was introduced in 1977 by Oldrich Vasicek.

<sup>29</sup> The Hull-White model was introduced in 1990 by John Hull and Alan White. The Vasicek model is a derived form of the Hull-White model.



$$dr_R(t) = (\theta_R(t) + b_{R\omega}\omega(t) - a_R r_R(t))dt + \sigma_R dW_R(t) \quad (3)$$

where  $W_R = (W_R(t))_{t \geq 0}$  is a Wiener process. The mean reversion level is given by  $\frac{\theta_R(t) + b_{R\omega}\omega(t)}{a_R}$ .

Since the nominal short rate is defined as the sum of the real short rate and the inflation short rate, i.e.  $r = r_R + r_i$ , the dynamics of the nominal short rate can be deduced from (2) and (3).

Concerning the short rate credit spread, its dynamics is given by a three-factor Hull-White model, where one driving factor is the so-called uncertainty index  $u$ .

$$du(t) = (\theta_u - a_u u(t))dt + \sigma_u dW_u(t) \quad (4)$$

$$ds(t) = (\theta_s + b_{su}u(t) - b_{s\omega}\omega(t) - a_s s(t))dt + \sigma_s dW_s(t) \quad (5)$$

where  $W_u = (W_u(t))_{t \geq 0}$  and  $W_s = (W_s(t))_{t \geq 0}$  are Wiener processes. The mean reversion level is given by  $\frac{\theta_s + b_{su}u(t) - b_{s\omega}\omega(t)}{a_s}$ .

Cascade 3 deals with the equity and the alternatives indexes. The dynamics of the stock return  $r_E$  is given by the following stochastic differential equation:

$$dr_E(t) = (\alpha_E + b_{E\omega}\omega(t) - b_{Ei}i(t) + b_{ER}r_R(t))dt + \sigma_E dW_E(t) \quad (6)$$

where  $W_E = (W_E(t))_{t \geq 0}$  is a Wiener process.

The dividend yield  $r_D$  dynamics is given by a Vasicek model, which means again that the property of mean reversion is being kept:

$$dr_D(t) = (\theta_D - a_D r_D(t))dt + \sigma_D dW_D(t) \quad (7)$$

where  $W_D = (W_D(t))_{t \geq 0}$  is a Wiener process. The mean reversion level is given by  $\frac{\theta_D}{a_D}$ .

### 7.1.3 Characteristics of Real World Scenarios

The long-term assumption on risk-return characteristics for different asset classes are based on equilibrium models and historical statistical analysis for the long investment horizons. The parameters entering into the scenario model are based on (derived from)

data and expert forecasts. The following Figures show the stochastic nature of the interest rate term structure at retirement age, the dynamics of the 1 year zero rate, the dynamics of the 5 years zero rate and the 10 years zero rate.

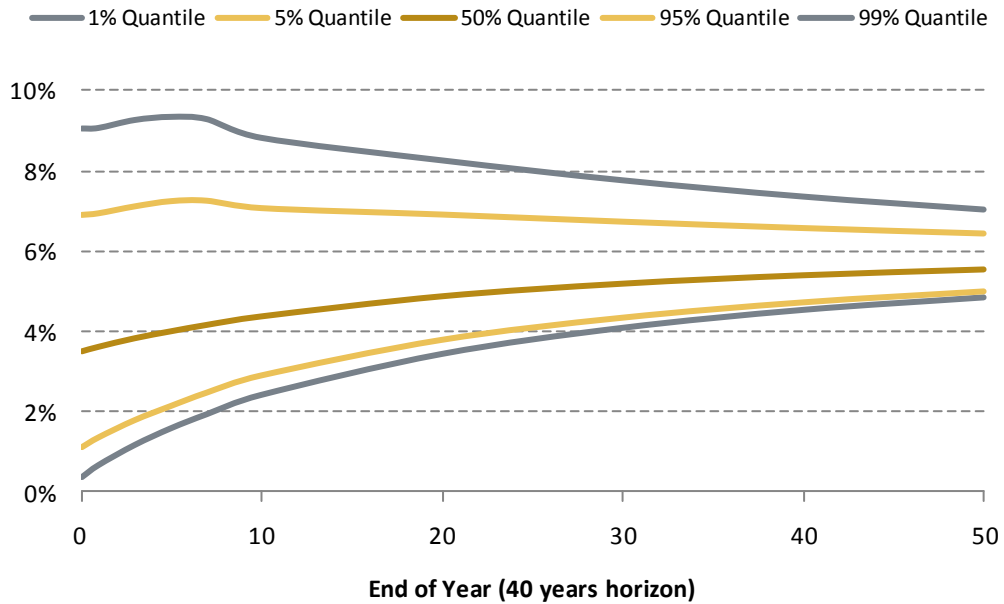


Figure 35: Term structure of interest rates at retirement age

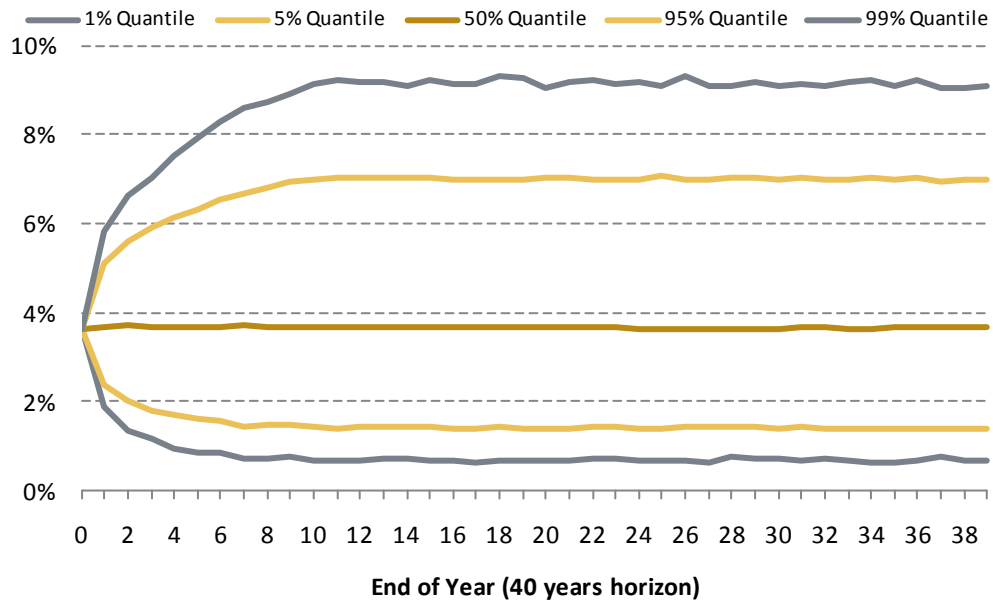


Figure 36: Dynamics of the 1 year zero rate

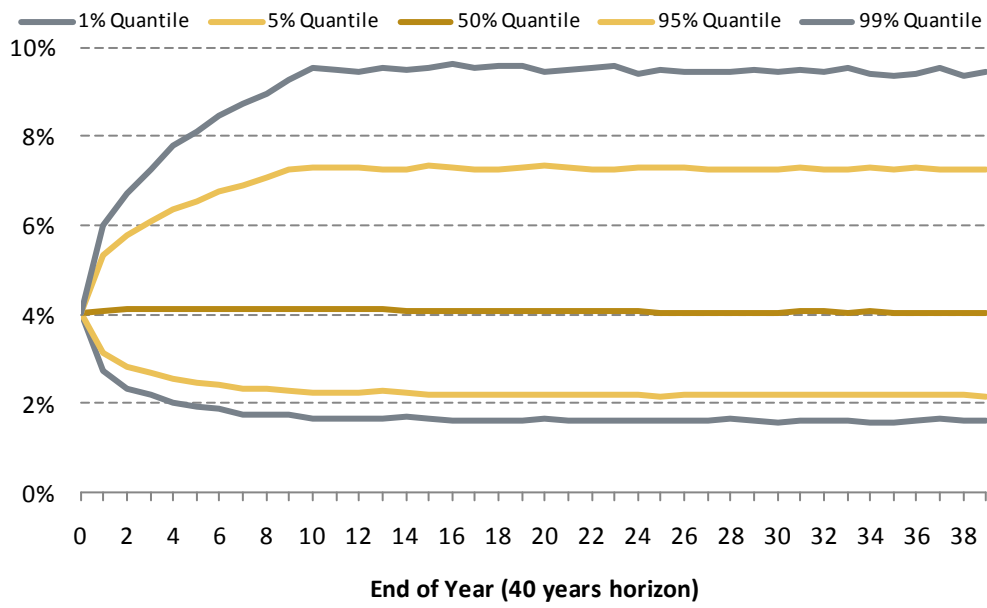


Figure 37: Dynamics of the 5 years zero rate

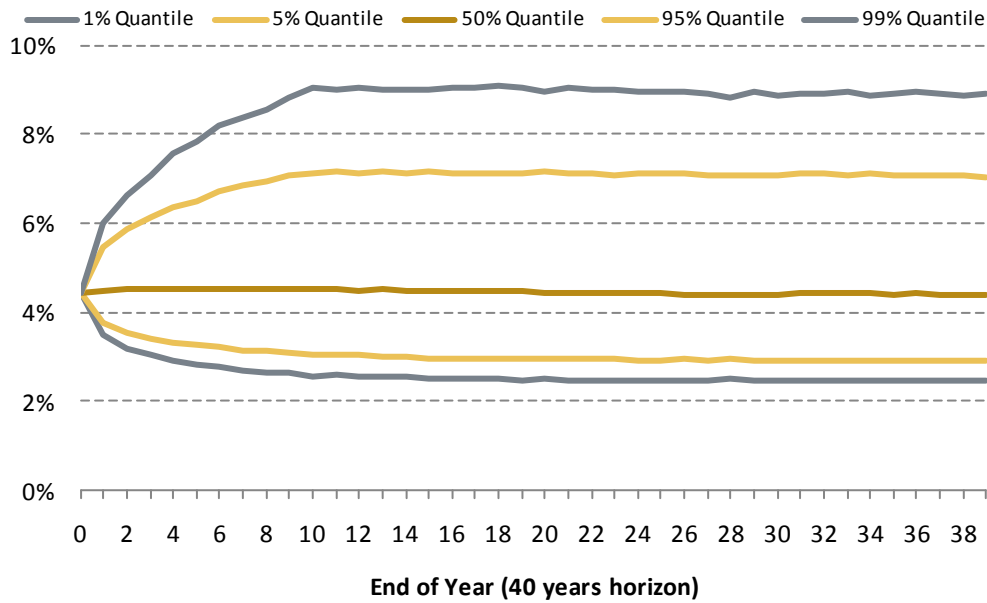


Figure 38: Dynamics of the 10 years zero rate

The following parameter assumptions on return and volatility were (implicitly) made for the different asset classes:

	Return	Volatility
Equity (*)	7.5%	20.0%
Government bonds (**)	4.8%	3.0%

(\*) based on the MSCI World Index.

(\*\*) portfolio duration of 5.6 years.

Table 17: Risk return characteristics of underlying asset classes

## 7.2 Sensitivity Analysis of Guarantee Fees

In the following, we will address once more the sensitivity of the guarantee fees to changes in the market environment. This time we focused on real market scenarios in the past. Different interest rate term structures, inflation rate term structures and levels of volatility can be found in the past and they have a huge impact on the prices of the guarantees as can be seen in the sensitivity analysis in Subsection 3.4. Figure 39 gives an overview of the historic market parameters and Table 18 characterizes these events.

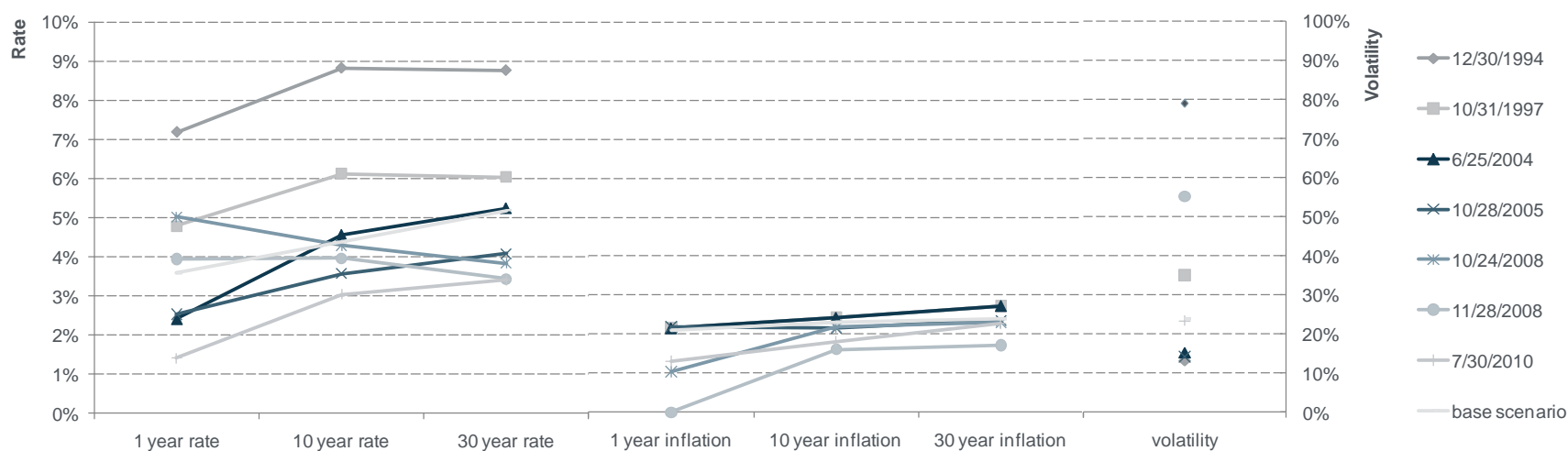


Figure 39: Overview of historical market scenarios

scenario	Interest Rates	Inflation Expectations	Volatility
12/30/1994	Very High	Medium	Low
10/31/1997	High	Medium	High
6/25/2004	Medium	Medium	Low
10/28/2005	Medium	High	Low
10/24/2008	Medium, Inverted	Medium	Very High
11/28/2008	Medium, Inverted	Low	High
7/30/2010	Low	Medium	Medium
base scenario	Medium	Medium	Medium

Table 18: Characteristics of historical market scenarios

scenario	1 year rate	10 year rate	30 year rate	1 year inflation	10 year inflation	30 year inflation	volatility
12/30/1994	7.19%	8.84%	8.78%	2.16%	2.42%	2.72%	13.20%
10/31/1997	4.80%	6.14%	6.05%	2.16%	2.42%	2.72%	35.09%
6/25/2004	2.42%	4.57%	5.24%	2.16%	2.42%	2.72%	15.19%
10/28/2005	2.54%	3.58%	4.09%	2.19%	2.16%	2.36%	14.25%
10/24/2008	5.04%	4.31%	3.84%	1.04%	2.18%	2.30%	79.13%
11/28/2008	3.95%	3.97%	3.44%	0.00%	1.61%	1.72%	55.28%
7/30/2010	1.42%	3.05%	3.42%	1.31%	1.81%	2.28%	23.50%
base scenario	3.60%	4.40%	5.20%	2.10%	2.30%	2.40%	24.00%

Table 19: Market parameters of historical scenarios and the base case

The impact of those different market scenarios on the guarantee fees can be depicted from Table 20. Prices for G\_4 for example vary from 0.6% to 4.6% in the different market environments. The changes are driven by interest rates, inflation and volatilities.

scenario	G_0	G_2	G_4	G_float	G_real	G_ongoing	G_HaircutEnd	G_HaircutOngoing
12/30/1994	0.0005%	0.0042%	0.0304%	0.6745%	0.0046%	0.0162%	0.8631%	0.0436%
10/31/1997	0.0562%	0.1829%	0.6000%	1.7241%	0.2050%	0.5682%	14.9661%	0.8116%
6/25/2004	0.0560%	0.2684%	1.3207%	0.6278%	0.3145%	0.2795%	49.8298%	4.0606%
10/28/2005	0.0554%	0.2796%	1.5351%	0.7460%	0.3376%	0.2100%	45.5068%	4.2479%
10/24/2008	0.3100%	0.6627%	1.5716%	5.7055%	0.4854%	1.4089%	31.2039%	1.4392%
11/28/2008	0.2445%	0.6122%	1.7208%	3.2940%	0.2442%	1.1334%	30.4826%	1.8133%
7/30/2010	0.2129%	0.8170%	4.6260%	1.0595%	0.5523%	0.7912%	94.8746%	16.4794%
base scenario	0.0550%	0.2180%	0.8870%	1.2240%	0.2410%	0.3890%	24.0640%	1.5990%

Table 20: Guarantee fees in historical market scenarios

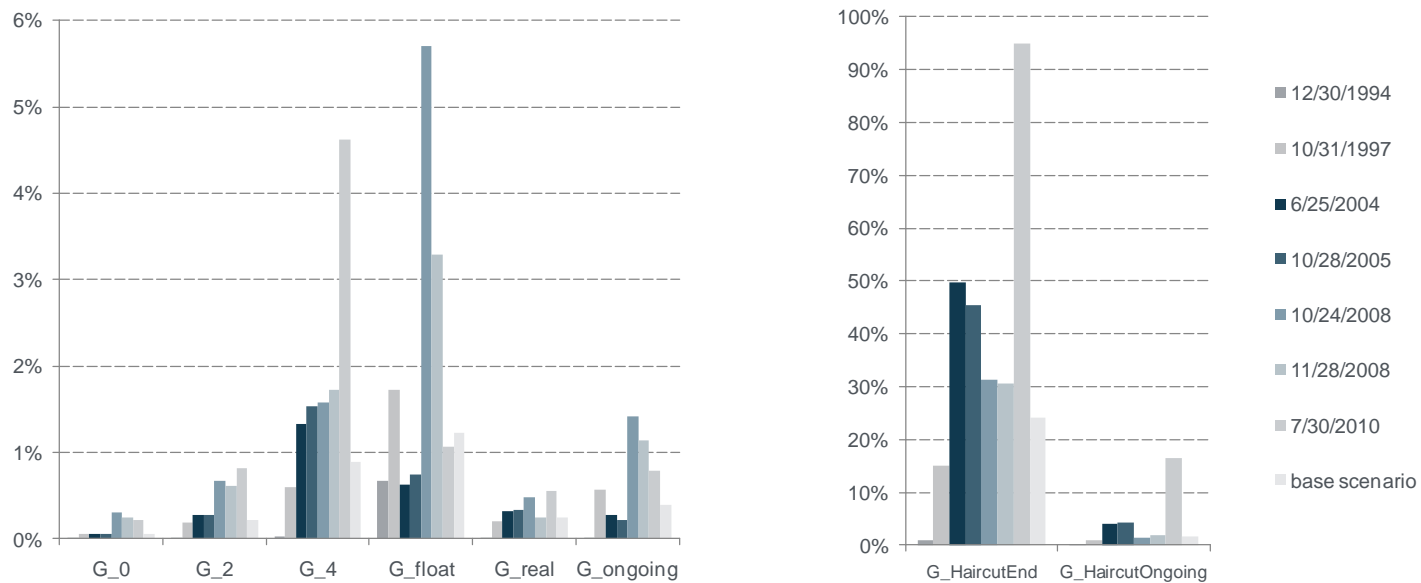


Figure 40: Comparison of guarantee fees in historical market scenarios



### 7.3 Replacement Rate Distributions

In this subsection, statistics on replacement rate distributions are provided for all the guarantee concepts given an underlying investment strategy and a defined savings period.

Investment Strategy: LC 80

Savings Period: 40 years

	Replacement Rate								
	no_G	G_0	G_2	G_4	G_real	G_ongoing	G_float	G_ongoing_haircut	G_final_haircut
	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80
1%-Q	22.65%	22.62%	26.56%	35.35%	27.85%	24.54%	29.83%	35.96%	36.01%
5%-Q	29.97%	29.66%	30.77%	38.97%	30.87%	30.76%	33.54%	40.09%	40.27%
25%-Q	47.72%	47.15%	45.49%	46.57%	45.26%	46.11%	43.68%	49.31%	49.51%
50%-Q	68.35%	67.49%	64.99%	56.83%	64.63%	64.67%	56.62%	63.20%	63.24%
75%-Q	100.39%	99.01%	95.13%	80.77%	94.60%	94.24%	79.60%	88.19%	88.03%
95%-Q	184.19%	181.51%	173.48%	145.00%	172.39%	169.38%	140.25%	150.63%	152.41%
99%-Q	287.82%	283.44%	270.96%	224.23%	269.32%	263.15%	218.27%	223.22%	232.66%
1%-CVaR	20.01%	20.72%	25.23%	33.76%	26.99%	22.55%	28.27%	34.42%	34.50%
5%-CVaR	25.56%	25.51%	28.19%	36.73%	29.04%	26.99%	31.16%	37.57%	37.68%
95%-CVaR	73.73%	72.77%	70.24%	63.80%	69.89%	69.84%	61.56%	68.83%	68.87%
99%-CVaR	79.64%	78.58%	75.77%	68.19%	75.38%	75.21%	65.80%	73.20%	73.36%
Mean	82.62%	81.52%	78.56%	70.45%	78.15%	77.91%	68.12%	75.38%	75.63%
STD	54.93%	54.04%	51.29%	39.69%	50.91%	49.81%	40.11%	40.18%	41.11%

Table 21: Statistics on replacement rate distributions for the investment strategy LC 80 and a savings period of 40 years

Investment Strategy: LC 50

Savings Period: 40 years

	Replacement Rate									
	no_G	G_0	G_2	G_4	G_real	G_ongoing	G_float	G_ongoing_haircut		G_final_haircut
	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80		LC 80
1%-Q	26.95%	26.80%	28.10%	35.33%	28.90%	26.85%	30.24%	36.13%		36.30%
5%-Q	33.98%	33.79%	33.32%	39.00%	33.14%	33.65%	34.06%	40.19%		40.29%
25%-Q	49.35%	49.04%	47.83%	46.56%	47.66%	48.18%	44.15%	49.26%		49.55%
50%-Q	64.67%	64.26%	62.61%	55.84%	62.39%	62.96%	55.53%	60.58%		60.81%
75%-Q	87.39%	86.78%	84.43%	73.65%	84.10%	84.98%	73.50%	79.12%		78.94%
95%-Q	144.39%	143.41%	139.46%	120.52%	138.90%	139.97%	120.05%	123.19%		124.25%
99%-Q	208.14%	206.59%	200.46%	173.18%	199.60%	201.60%	178.21%	174.38%		177.12%
1%-CVaR	24.28%	24.21%	26.34%	33.76%	27.88%	24.50%	28.70%	34.58%		34.69%
5%-CVaR	29.70%	29.55%	30.01%	36.75%	30.41%	29.47%	31.60%	37.70%		37.82%
95%-CVaR	67.77%	67.34%	65.66%	60.17%	65.43%	65.97%	58.62%	63.95%		64.09%
99%-CVaR	71.82%	71.36%	69.55%	63.38%	69.31%	69.87%	61.94%	67.10%		67.30%
Mean	73.90%	73.42%	71.55%	65.06%	71.30%	71.89%	63.88%	68.70%		68.96%
STD	39.19%	38.90%	37.64%	30.26%	37.45%	37.79%	33.11%	30.07%		30.63%

Table 22: Statistics on replacement rate distributions for the investment strategy LC 50 and a savings period of 40 years

Investment Strategy: LC 20

Savings Period: 40 years

	Replacement Rate									
	no_G	G_0	G_2	G_4	G_real	G_ongoing	G_float	G_ongoing_haircut		G_final_haircut
	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80		LC 80
1%-Q	32.12%	32.07%	31.73%	34.82%	31.67%	31.99%	31.21%	36.06%		36.10%
5%-Q	37.30%	37.24%	36.81%	38.42%	36.74%	37.14%	35.11%	39.56%		39.60%
25%-Q	47.03%	46.96%	46.38%	44.99%	46.29%	46.83%	43.69%	46.96%		47.02%
50%-Q	56.93%	56.84%	56.14%	51.85%	56.03%	56.69%	52.48%	54.94%		55.00%
75%-Q	71.50%	71.38%	70.45%	63.74%	70.30%	71.18%	65.83%	67.21%		67.43%
95%-Q	116.73%	116.53%	114.93%	103.25%	114.68%	116.16%	107.93%	104.03%		105.44%
99%-Q	193.24%	192.89%	190.17%	170.36%	189.72%	192.41%	175.31%	162.42%		168.57%
1%-CVaR	30.25%	30.20%	29.92%	33.32%	30.19%	30.13%	29.96%	34.38%		34.45%
5%-CVaR	34.25%	34.20%	33.82%	36.24%	33.83%	34.11%	32.82%	37.30%		37.34%
95%-CVaR	59.29%	59.19%	58.45%	54.62%	58.33%	59.03%	54.88%	57.15%		57.32%
99%-CVaR	62.56%	62.46%	61.67%	57.43%	61.54%	62.29%	57.88%	59.81%		60.07%
Mean	64.68%	64.57%	63.75%	59.25%	63.62%	64.39%	59.85%	61.46%		61.83%
STD	33.77%	33.70%	33.19%	28.58%	33.10%	33.60%	31.15%	26.75%		28.04%

Table 23: Statistics on replacement rate distributions for the investment strategy LC 20 and a savings period of 40 years

Investment Strategy: LC 80

Savings Period: 20 years

Replacement Rate									
	no_G	G_0	G_2	G_4	G_real	G_ongoing	G_float	G_ongoing_haircut	G_final_haircut
	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80
1%-Q	9.28%	9.21%	9.74%	11.02%	10.02%	9.40%	10.26%	11.26%	11.23%
5%-Q	10.58%	10.36%	10.60%	11.74%	10.68%	10.49%	10.97%	12.00%	11.98%
25%-Q	13.34%	13.03%	12.34%	12.88%	12.41%	12.75%	12.25%	13.25%	13.18%
50%-Q	15.95%	15.56%	14.58%	13.83%	14.64%	15.00%	13.57%	14.33%	14.18%
75%-Q	19.50%	18.99%	17.71%	15.06%	17.78%	18.11%	15.56%	15.79%	15.50%
95%-Q	27.22%	26.43%	24.57%	18.07%	24.70%	25.02%	20.60%	18.95%	18.28%
99%-Q	35.14%	34.08%	31.61%	22.08%	31.75%	32.08%	26.61%	22.85%	21.28%
1%-CVaR	8.63%	8.72%	9.44%	10.70%	9.73%	8.94%	9.96%	10.92%	10.92%
5%-CVaR	9.77%	9.66%	10.09%	11.30%	10.27%	9.82%	10.54%	11.54%	11.52%
95%-CVaR	16.23%	15.84%	14.96%	13.90%	15.03%	15.30%	13.87%	14.43%	14.25%
99%-CVaR	16.79%	16.38%	15.45%	14.13%	15.53%	15.80%	14.23%	14.67%	14.46%
Mean	17.03%	16.61%	15.66%	14.24%	15.74%	16.01%	14.40%	14.78%	14.55%
STD	5.38%	5.20%	4.63%	2.17%	4.64%	4.72%	3.32%	2.31%	2.03%

Table 24: Statistics on replacement rate distributions for the investment strategy LC 80 and a savings period of 20 years

## 7.4 Further Statistics

Table 25 outlines the probability that the guarantee is activated at least one time during the savings phase. For the guarantees G\_0, G\_2, G\_4, G\_real, G\_float, G\_ongoing\_haircut and G\_ongoing\_haircut, the analysis focuses on the retirement age whereas for G\_ongoing, the focus is on the whole savings phase.

	Probability that the guarantee is activated at least once							G_ongoing_haircut	G_final_haircut
	G_0	G_2	G_4	G_real	G_ongoing	G_float			
LC 80, IH 40Y(*)	0.49%	5.75%	35.32%	6.48%	83.45%	40.33%	23.09%	21.26%	
LC 50, IH 40Y(*)	0.06%	2.12%	30.41%	2.56%	66.25%	33.27%	19.15%	17.21%	
LC 20, IH 40Y(*)	0.00%	0.07%	30.17%	0.30%	24.53%	17.83%	16.32%	14.80%	
LC 80, IH 20Y(*)	0.99%	13.81%	86.49%	14.35%	82.85%	76.98%	39.93%	26.95%	

(\*) IH: Investment Horizon

Table 25: Probability that the guarantee is activated at least one time for all the investment strategies during different savings period

The results shown in Table 26 represent the probability that the lump sum achieved by the different guarantees is higher than the lump sum obtained when no guarantee option is selected.

	Probability that the lump sum is higher than the lump sum obtained without guarantee							G_ongoing_haircut	G_final_haircut
	G_0	G_2	G_4	G_real	G_ongoing	G_float			
LC 80, IH 40Y(*)	0.48%	4.78%	21.26%	5.22%	18.20%	21.72%	21.26%	21.26%	
LC 50, IH 40Y(*)	0.06%	1.68%	17.21%	2.00%	6.96%	16.24%	17.21%	17.21%	
LC 20, IH 40Y(*)	0.00%	0.07%	14.80%	0.24%	0.39%	7.78%	14.80%	14.80%	
LC 80, IH 20Y(*)	0.74%	6.92%	26.95%	7.79%	14.38%	24.52%	26.95%	26.95%	

(\*) IH: Investment Horizon

Table 26: Probability that the lump sum achieved with the guarantee is higher than the lump sum achieved with no guarantee for all the investment strategies during different savings period

The cost of the guarantee can be defined as a loss occurred when comparing the lump sum achieved at retirement age where no guarantee option is selected and the lump sum achieved if a guarantee option is chosen. Given that definition, the cost measure would be expressed as a percentage loss, where for each scenario the difference introduced previously is divided by the lump sum achieved where no guarantee option is selected:

$$\text{Guarantee Cost}_s = \frac{1}{\text{Lump Sum}_s^{\text{no Guarantee}}} (\text{Lump Sum}_s^{\text{no Guarantee}} - \text{Lump Sum}_s^{\text{with Guarantee}}), \forall s \in \{1, \dots, 10000\}$$

Table 27 summarizes the outcomes for our base case (i.e. for the investment strategy LC80 and a savings phase of 40 years).

	Cost of Guarantee								
	no_G	G_0	G_2	G_4	G_real	G_ongoing	G_float	G_ongoing_haircut	G_final_haircut
	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80	LC 80
Mean	0.00%	1.22%	3.86%	7.11%	4.22%	4.08%	10.05%	0.96%	0.84%
1%-Q	0.00%	0.90%	-29.74%	-93.61%	-31.27%	-28.52%	-110.29%	-93.61%	-93.61%
5%-Q	0.00%	1.03%	0.83%	-47.67%	-0.71%	-11.93%	-51.49%	-47.67%	-47.67%
25%-Q	0.00%	1.18%	4.58%	5.69%	5.04%	2.57%	5.22%	1.45%	1.37%
50%-Q	0.00%	1.28%	4.98%	18.30%	5.49%	7.14%	23.81%	6.99%	7.67%
75%-Q	0.00%	1.38%	5.36%	19.88%	5.91%	8.69%	26.13%	12.55%	12.65%
95%-Q	0.00%	1.52%	5.89%	21.75%	6.49%	9.93%	28.62%	19.83%	17.47%
99%-Q	0.00%	1.60%	6.19%	22.80%	6.82%	10.57%	29.95%	24.05%	19.83%
1%-CVaR	0.00%	-5.06%	-47.66%	-120.08%	-51.34%	-38.49%	-147.36%	-120.08%	-120.08%
5%-CVaR	0.00%	-0.23%	-17.86%	-75.81%	-20.00%	-21.93%	-88.01%	-75.81%	-75.81%
95%-CVaR	0.00%	1.20%	3.74%	6.31%	4.09%	3.75%	9.03%	-0.17%	-0.11%
99%-CVaR	0.00%	1.22%	3.84%	6.95%	4.19%	4.01%	9.84%	0.71%	0.64%

Table 27: Comparison of guarantee costs